The conception / handling of infiniteness

The “infinitesimal” (algebra centric) conception (Euler-Leibniz) vs. the “puristic”analysis centric conception (Riemann)

Extract from
Vita Mathematica
Bernhard Riemann, 1826-1866

Detlef Laugwitz
Birkhäuser Verlag, 1996

Abstract:

Section 4.2 of the mentioned book is about the “turning point in the conception of the infiniteness in mathematics”, based on the work of B. Riemann and his friend G. L. Dirichlet. We recall the simple example of this section which highlights the “validity of the Euler-Leibniz conception, which has beed acknowledged in the last century very well, while the foundation of Einstein’s gravity theory is still based on the “conservative” beliefs of B. Riemann.

The series ln(2)

The series

(1) $$\ln 2 = 1 - \frac{1}{2} + \frac{1}{3} - \frac{1}{4} + \frac{1}{5} - \frac{1}{6} + -$$

is based on the power series development of the function $\ln(1+x)$, while the series

(2) $$\frac{3}{2}\ln 2 = 1 + \frac{1}{3} - \frac{1}{2} + \frac{1}{5} - \frac{1}{7} - \frac{1}{4} + + -$$

are based on the power series development of the function $\frac{1}{2}\ln(1+x^2)\ln(1+x)^2$, $(x=1)$.

According to Euler-Leibniz conceptions both values can be proven based on the Zeta function series assuming a numerous infinite integer value $i \equiv \infty$ (!!):

$$\zeta(1) = \sum_1^\infty \frac{1}{n} = \sum_1^i \frac{1}{n} = 1 + \frac{1}{2} + \frac{1}{3} + \frac{1}{4} + \frac{1}{5} + \frac{1}{6} + ... + \frac{1}{i} = \ln(i) + \gamma_i$$

whereby γ_i (in modern Non-Standard analysis terms) denotes a “ideal point” with standard value $st(\gamma_i) = \gamma$, being the Euler constant. Already Euler applied the following conception to prove the “values” above.The “philosophy” behind this (perception/conception) can be interpreted as a sort of Leibniz’s continuity principles, leading to the following equivalences:

(1) $$1 - \frac{1}{2} + \frac{1}{3} - \frac{1}{4} + ... + \frac{1}{4i-1} - \frac{1}{4i} = 1 + \frac{1}{2} + ... + \frac{1}{2i}) - 2(\frac{1}{2} + \frac{1}{4} + ... + \frac{1}{2i}) =$$

$$= \ln(2i) + \gamma_{2i} - (\ln(i) + \gamma_i) \approx \ln(2i) - \ln(i) = \ln(2)$$

(2) $$1 + \frac{1}{3} - \frac{1}{2} + \frac{1}{5} + \frac{1}{7} - \frac{1}{4} .... + \frac{1}{4i-3} - \frac{1}{4i-1} - \frac{1}{2i} = 1 + \frac{1}{2} + ... + \frac{1}{2i}) =$$

$$= 1 + \frac{1}{2} + \frac{1}{3} + ... + \frac{1}{4i} - (\frac{1}{2} + \frac{1}{4} + \frac{1}{6} .... + \frac{1}{4i}) - (\frac{1}{2} + \frac{1}{4} + \frac{1}{6} .... + \frac{1}{2i}) =$$

$$= \ln(4i) + \gamma_{4i} + \frac{1}{2}(\ln(2i) + \gamma_{2i}) - \frac{1}{2}(\ln(i) + \gamma_i) \approx \ln(4) - \frac{1}{2}\ln(2) = \frac{3}{2}\ln(2)$$