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Energy gradient method for turbulent transition with consideration of effect of disturbance frequency

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ABSTRACT: The energy gradient theory for flow instability and turbulent transition was proposed in our previous work. It was shown that the disturbance amplitude required for turbulent transition is inversely proportional to Re, which is in agreement with the experiments. In present study, the energy gradient theory is extended to include the effect of disturbance frequency on turbulent transition. The theoretical result obtained accords well with the experimental data in literature.

KEY WORDS: Turbulent Transition; Energy gradient method; Disturbance amplitude; Disturbance frequency.

1 INTRODUCTION

Turbulent transition is still a great challenge in fluid dynamics and fluid physics even though it has been a topic of intense investigation for more than a century. In the past, several stability theories have been proposed to describe or predict the onset of turbulent transition. These include: (1) The linear stability theory [1], which goes back to Rayleigh [2], has been used to try to explain flow instability and turbulent transition. However, it is only successful for predicting the primary instabilities in a few cases such as Taylor-Couette flow between rotating concentric cylinders and Rayleigh-Bernard heat convection on flat plate, and it is failed for wall-bounded parallel flows, such as plane Couette flow, plane Poiseuille flow and pipe Poiseuille flow (see Fig.1). (2) The energy method based on the Reynolds-Orr equation is another mature method for estimating the increase of disturbance energy[3]. However, agreement could not be obtained between the theoretical predictions and the experiment data for the mentioned wall bounded flows. (3) The weakly nonlinear stability theory (Stuart, 1960) emerged in the last century (see [4]). (4) The secondary instability theory (Herbert et al, 1988) was developed most recently, and it explains some of flow transition phenomena better than the other earlier theories (see [5]). However, there are still significant discrepancies between the predictions obtained using this method and experimental data; particularly at transition.

Lin obtained the first asymptotic solution for linear instability in plane Poiseuille flow and proved that this type of flow is unstable at Re of about 8000 [6]. However, as suggested, a linearized theory of hydrodynamic stability is of course not quite adequate to account for transition to turbulence [6]. For wall bounded parallel flows, it is observed from experiments that there is a critical Reynolds number Re_c below which no turbulence can be sustained regardless of the level of imposed disturbance. For the pipe Poiseuille flow, this critical value of Reynolds number is about 2000 from experiments [7-11]. Above this Re_c, the transition to turbulence depends to a large extent on the initial disturbance to the flow, besides the value of Re of the flow. For example, experiments showed that if the disturbances in a laminar flow can be carefully avoided or considerably reduced, the onset of turbulence can be delayed to Reynolds number up to Re=O(10^5) [7-11]. Experiments also showed that for $Re > Re_c$, only when a threshold of disturbance amplitude is reached, can the flow transition to turbulence occur [10-11]. Trefethen et al. [12] suggested that the critical amplitude of the disturbance leading to transition varies broadly with the Reynolds number and is associated with an exponent rule of the form, $A \propto Re^{\gamma}$. The magnitude of this exponent has significant implication for turbulence research.

Recently, Dou [13-14] proposed an energy gradient method with the aim of clarifying the mechanism of flow transition from laminar to turbulence. He gave detailed derivations for this method based on the flow physics. For plane Poiseuille flow and Hagen-Poiseuille flow, this method yields results consistent with experimental data. It also works well for boundary layer flow and Taylor-Couette flow [15-17]. It was shown that the disturbance amplitude required for turbulent transition is inversely proportional to Re, which is in agreement with the experiments and simulations [10-11, 18-19]. However, experiments and numerical simulations showed that the disturbance amplitude at given Re is also influenced by the disturbance frequency [9, 20-21]. In present study, the energy gradient theory is extended to include the effect of disturbance frequency on turbulent transition. Then, the theoretical result will be compared with the available experimental data.

2 ENERGY GRADIENT METHOD

Dou [13-14] proposed a mechanism with the aim to clarify the phenomenon of transition from laminar flow to turbulence for wall-bounded shear flows. This theory was derived from Newtonian mechanics considering the base flow subjected to a disturbance with periodic variation. In this mechanism, the whole flow field is treated as an energy field. It is suggested that the gradient of total mechanical energy in the transverse direction of the main flow has the potential to amplify a velocity disturbance, while the viscous friction loss of the total mechanical energy in the streamwise direction can resist and absorb this disturbance. The flow instability or the transition to turbulence depends on the relative magnitude of these two roles of energy gradient amplification and viscous friction damping of the initial disturbance.

For a given base flow, the fluid particles may move in an oscillatory fashion along the streamwise direction if they are subjected to a disturbance (Fig.2(a)). With the oscillatory motion, the fluid particle may gain or lose energy (ΔE) via the disturbance, and simultaneously this particle may have energy loss (ΔH) due to the fluid viscosity along the streamline direction. The analysis suggests that the magnitudes of ΔE and ΔH determine the stability of the flow of fluid particles when the flow is subjected to a periodic disturbance.

Thus, the relative magnitude of the energy of fluid particles gained and the energy loss due to viscous friction in a disturbance cycle determines the disturbance amplification or decay and gives rise to the stability criterion. For a given flow, a stability criterion is written as below for a half-period [13,14],

$$F = \frac{\Delta E}{\Delta H} = \left(\frac{\partial E}{\partial n} \frac{2A}{\pi} \right) \Bigg/ \left(\frac{\partial H}{\partial s} \frac{\pi}{\omega} u \right) = \frac{2}{\pi^2} K \frac{A\omega}{u},$$
$$= \frac{2}{\pi^2} K \frac{v'_m}{u} < \text{Const} \quad (1)$$

and

$$K = \frac{\partial E / \partial n}{\partial H / \partial s}. \quad (2)$$

Here, $\Delta E = \frac{\partial E}{\partial y} \frac{2A}{\pi}$ is the energy variation of per unit volume of fluid for a half-period, $\Delta H = \frac{\partial H}{\partial x} l = \frac{\partial H}{\partial x} \frac{\pi}{\omega} u$ is the energy loss along the streamline per unit volume of fluid for the first half-period, and $l = u(T/2) = u(\pi/\omega)$ is the streamwise length moved by the particle in a half-period. F is a function of space which expresses the ratio of the energy gained in a half-period by the particle (ΔE) and the energy loss due to viscosity in the half-period (ΔH). K is a dimensionless field variable (function) and expresses the ratio of transversal energy gradient and the rate of the energy loss along the streamline, which can be calculated from Navier-Stokes equations. $E = (1/2)\rho V^2$ is the kinetic energy per *unit volumetric fluid*, s is along the streamwise direction and n is along the transverse direction. H is the energy loss per *unit volumetric fluid* along the streamline for finite length. Furthermore, ρ is the fluid density, u is the streamwise velocity of the main flow, $v'_m = A\omega$ is the disturbance amplitude of velocity and the disturbance has a period of $T = 2\pi/\omega$, A is the amplitude of disturbance in the transverse direction, and ω is the frequency of the disturbance.

In Eq.(1), when ΔE is large and ΔH is small, F will be very large. When F reaches a magnitude larger than a critical value, the flow will be unstable. Otherwise, the flow is stable and keeps to being laminar. Therefore, it can be suggested from Eq.(1) that the instability of a flow depends on the values of K and the amplitude of the relative disturbance velocity v'_m/u. For all types of flows, it has been shown that the magnitude of K is proportional to the global

Reynolds number for a given geometry [13-15]. Thus, the criterion of Eq.(1) can be written as,

$$Re\frac{v'_m}{u} < \text{Const} . \qquad (3)$$

For a given flow geometry, U is a characteristic velocity and generally is a function of u. Thus, Eq.(3) can be written as,

$$Re\frac{v'_m}{U} < \text{Const} , \qquad (4)$$

or $$(\frac{v'_m}{U})_c \sim (Re)^{-1} . \qquad (5)$$

This scaling has been confirmed by careful experiments for pipe flow in [10-11].

This theory obtains good agreement with the experiments in three aspects [13-17]. (1) The threshold amplitude of disturbance for transition to turbulence is scaled with Re by an exponent of -1 in parallel flows (Fig.3), which explains the recent experimental results of pipe flow by Hof et al. [10] and also Peixinho and Mullin [11] where injection disturbances are used. (2) For wall bounded parallel flows, turbulent transition takes place at a critical value of the energy gradient parameter, $K_{\max}$, about 370-380, below which no turbulence exists. (3) The location where the flow instability is first initiated accords with the experiments. This location is at y/h=0.58 for plane Poiseuille flow and at r/R=0.58 for pipe Poiseuille flow, which have been confirmed by Nishioka et al's [20] experiments and Nishi et al's [18] experiments, respectively.

3 ENERGY GRADIENT METHOD: CONSIDERING THE EFFECT OF DISTURBANCE FREQUENCY

In Dou [13,14], the formulations in energy gradient method were derived in an ideal model using a half period that the energy variation and the energy loss in this half period represent those in all periods. If we use a whole period, the formulations derived are the same (see below).

Using the same method, the energy variation of per unit volume of fluid for the first half-period is $\Delta E_1 = \frac{\partial E}{\partial y}\frac{2A}{\pi}$, and that for the second half period is $\Delta E_2 = -\frac{\partial E}{\partial y}\frac{2A}{\pi}$. Generally, the magnitudes of these two terms are not equal because the base flow is not uniform and the disturbance wave is distorted in its propagation downstream, see Fig.2(b), which is due to the difference of the propagation speed between the upper and the down half-period. The net energy gained, $\Delta E = \Delta E_1 + \Delta E_2$, is proportional to ΔE_1 or ΔE_2 and we may write as

$$\Delta E = \xi \frac{\partial E}{\partial y}\frac{2A}{\pi} \qquad (6)$$

where ξ is a constant. The energy loss per unit volume of fluid along the streamline for the whole period is

$$\Delta H = \frac{\partial H}{\partial x} 2l = 2\frac{\partial H}{\partial x}\frac{\pi}{\omega}u . \qquad (7)$$

Assuming that a characteristic streamwise length to be observed for disturbance development is L, see Fig.4. Then the number of the waves in this length is $n = \frac{L}{u}\frac{\omega}{2\pi}$, since the propagation speed in the injection disturbance is equal to the flow velocity. For a given base flow, a stability criterion is written as follow for the observed length,

$$F = n\frac{\Delta E}{\Delta H} = \frac{L}{u}\frac{\omega}{2\pi}\cdot\left(\xi\frac{\partial E}{\partial n}\frac{2A}{\pi}\right)\Big/\left(2\frac{\partial H}{\partial s}\frac{\pi}{\omega}u\right) = \frac{4\xi}{\pi^2}K\frac{A\omega}{u}\frac{L}{u}\frac{\omega}{2\pi} = \frac{2\xi}{\pi^3}K\frac{v'_m}{u}\frac{\omega L}{u} < \text{Const} , \qquad (8)$$

and

$$K = \frac{\partial E/\partial n}{\partial H/\partial s} . \qquad (9)$$

From above equation, it can be obtained that the relation of the disturbance amplitude and the frequency at turbulent transition for a given base flow and the observed length is,

$$\omega^2 A = \omega v'_m = \text{Const} . \qquad (10)$$

It is known that the average kinetic energy for a periodic disturbance is proportional to $\omega^2 A^2$. It can be seen that the disturbance energy is not a constant for turbulent transition when the disturbance amplitude or frequency is allowed to vary for given base flow.

4 COMPARISON WITH EXPERIMENTAL DATA

Darbyshare and Mullin [9] carried out extensive experiments on finite amplitude disturbance induced turbulent transition in a pipe with injection disturbance on the wall. In their experiment, the disturbance is introduced into the inlet section of the pipe by means of jet(s) which is associated with the piston movement. The piston is driven by a mechanical mechanism with a periodic motion. The velocity disturbance due to the movement of the piston is a form of pulse-linear disturbance. The amplitude of distance of the piston motion is A, the disturbance frequency is ω, and thus the amplitude of the disturbance velocity in the pipe is ωA. Figure 5 shows the experimental result at Re=2200 for the disturbance in the form of a single jet injected orthogonally to the main stream flow through a small hole opening on the pipe wall. The experimental data in the figure shows the critical amplitude *A* of the piston versus the frequency of the injection. It is found that for a given Re, the critical amplitude *A* depends on the frequency of the disturbance, and the critical amplitude *A* decreases with the increasing frequency. This suggests that the disturbance energy may be relevant, since the average kinetic energy of the disturbance in a period is proportional to $\omega^2 A^2$. It is, therefore, further observed from these experimental data that the critical energy of disturbance is not a constant with the variation of frequency at a fixed Re. The theoretical result of present study is compared with the experimental results in Fig.6. It can be seen that the present theory, $\omega^2 A$ =constant (Eq.(10)), is excellently in agreement with the experiment.

Nishioka et al [20] did the first experiment that shows linear instability occurrence in a channel flow at a critical Reynolds number about Re=6000, and the exact solution for linear instability mentioned in their work is at Re=5771. They also checked the finite amplitude disturbance for turbulent transition at subcritical condition. They used a vibrating ribbon technique to produce a sinusoidal disturbance to the flow which is controlled by an exciting electric current. Their results showed that the disturbance frequency has a complicated influence on the threshold of disturbance amplitude. Their results are plotted using a relation of the amplitude of streamwise disturbance u_m'/U_c to the frequency $\omega(h/U_{c)}$ (Fig.6). According to the mass conservation, the maximum of u_m' corresponds to a maximum of v_m' at transition. Their frequencies are the same and the phase difference may vary along the transverse direction. Therefore, the variation of u_m' should also obey Eq.(10) if v_m' varies as present theory. The present theory is compared with their experimental data as shown in Fig.6. It can be seen that the agreement is deemed fairly reasonable at low frequency. At higher $\omega(h/U_{c)}$, u_m'/U_c first increases and then decreases with increasing $\omega(h/U_{c)}$. This phenomenon may be related to the three-dimensionality induced in higher $\omega(h/U_{c)}$. At a given u_m', v_m' will be reduced due to the effect of spanwise disturbance development, while the transition is controlled by v_m' in terms of present theory (see Eq.8). In order to reach the value of v_m' to trigger transition, u_m' must increase. As to the large decrease of u_m' at very large frequency, it appears that the transition is trigged directly by spot like fluctuation appearing abruptly before the fundamental has grown sufficiently, as Nishioka et al suggested [20].

5 MECHANISM OF TRANSITION AND ROLE OF DISTURBANCE

Based on the energy gradient method, criteria for turbulent transition have been proposed [17]. For pressure driven flows, it is demonstrated that the necessary and sufficient condition for turbulent transition is the presence of the velocity inflection point in the averaged flow (i.e., singularity of energy gradient function). It is shown that the role of disturbance in the transition is to cause the energy gradient function to approach the singularity (i.e., inflection of velocity profile).

According to the principle of energy gradient theory, the disturbance of velocity interacts with the base flow which causes the disturbed fluid particles to gain energy in periodic motion. The energy gained by fluid particles is accumulated with the fluid particles progressing downstream. This velocity disturbance is overlapped with the mean velocity, which makes the profile of mean velocity gradually distorted. When the initially input disturbance is sufficiently large, the development of the disturbance will result in the formation of inflection point on the mean velocity. Since the role of viscous damping to the disturbance at inflection point is zero, the appearance of velocity inflection will abruptly cause the flow breakdown to transit to turbulence.

In terms of the energy gradient theory, in the process of transition occurrence, the role of disturbance is to interact with the mean flow and to promote the formation of velocity inflection point. Wedin and Kerswell's [22] simulation with disturbance of travelling waves in pipe flow showed that the formation of inflection point is unavoidable leading to flow breakdown to turbulence. Wang et al [21] found in the simulation of plane Poiseuille flow that the variation of mean velocity profile with the disturbance development in time is a key feature. They also found that the variation of mean velocity profile with time makes itself more unstable gradually, and finally the flow breakdown occurs after the appearance of the inflection point. Hof et al [23] showed by experiment that velocity inflection is the core of turbulent puffs (spots) in pipe flows, which plays the role of transferring the energy from the mean shear into turbulent eddies. Cancelling (or removing) or reducing this inflection point, the turbulent spot/puff will be removed or delayed.

6 CONCLUSION

In this study, the energy gradient method is firstly introduced. Then, the energy gradient theory is extended to include the effect of disturbance frequency on turbulent transition. A set of formulations have been given to show the relation of disturbance amplitude with frequency at transition. The theoretical result obtained is compared with the available experimental data in literature, and reasonable agreement is achieved. This is shown perhaps for the first time that effect of disturbance frequency on turbulent transition is theoretically demonstrated. These results are helpful to understand the physical mechanism behind the turbulent transition with finite amplitude of disturbances.

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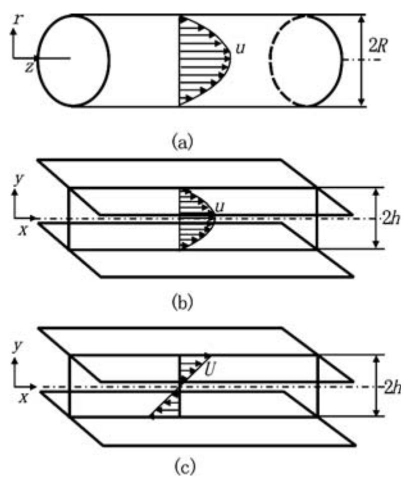

Fig.1 Schematic of wall bounded parallel flows. (a) Pipe Poiseuille flow; (b) Plane Poiseuille flow; (c) Plane Couette flow

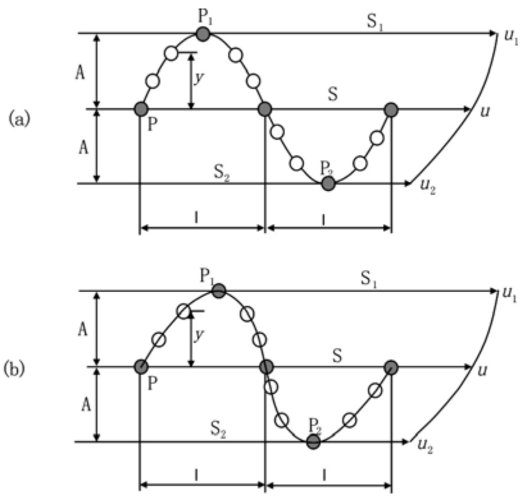

Fig.2 Movement of a particle around its original equilibrium position in a cycle of disturbance. (a) Ideal case; (b) Wave deformed

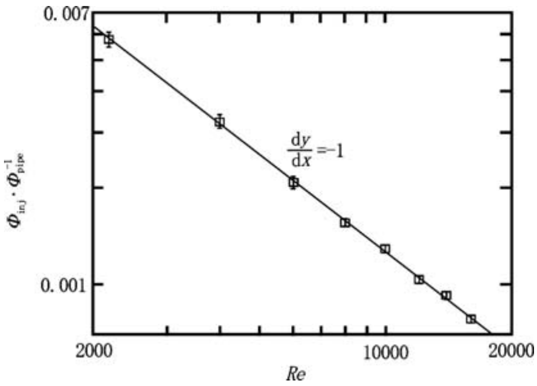

Fig.3 Experimental results for pipe flow: the normalized flow rate of disturbance versus the Reynolds number (Hof, Juel, and Mullin [10]). The range of Re is from 2000 to 18,000. The normalized flow rate of disturbance is equivalent to the normalized amplitude of disturbance for the scaling of Reynolds number, $\Phi_{inj} / \Phi_{pipe} \sim \left(v'_m / U\right)_c$

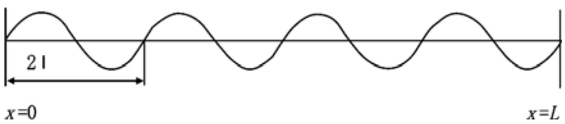

Fig.4 Length to be observed in the flow including multi-periods

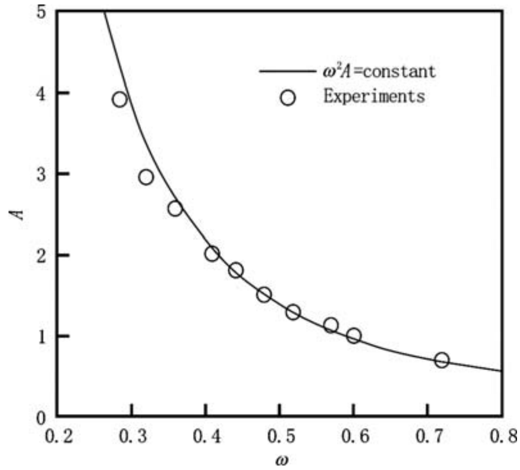

Fig. 5 Variation of critical disturbance amplitude, A, against the driving angular frequency ω at *Re*=2200 for the single-jet disturbance. Comparison of the present theory Eq.(10) with the experimental data in [9]

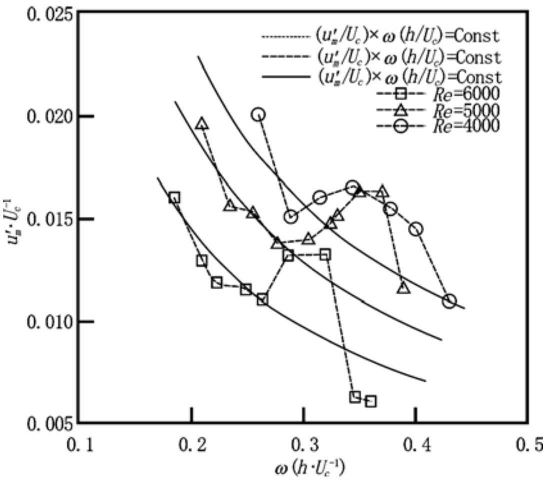

Fig. 6 Variation of critical amplitude of disturbance velocity, u_m'/U_c, against the disturbance frequency $\omega(h/U_c)$. Comparison of the present theory Eq.(10) with the experimental data in [20]. Here, *h* is the half channel width, ω is the angular frequency of disturbance, U_c is the centreline velocity, and u_m' is the disturbance amplitude of streamwise velocity