Thoughts about a Quantum Gravity Theory

Non-Standard Analysis, Distributions, Differential Forms and Variational Theory

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§ 1 Real and Hyper-real World and Field

Currently there are two one-way roads ending up in quantum theory and gravitation theory. Both together are inconsistent, i.e. going back “the road to reality” (R. Penrose) there must have been a branch, where underlying common sense assumptions lead to the two different ways forward ending up in today’s paradox and contradictionary physical models (of an empty, filled space).

Our thoughts are based on the following assumptions:

1. “... *modern physics is already itself mathematical...*” (M. Heidegger [MHe2])

2. “... *general relativity and quantum mechanics can be derived from a small set of postulates, one or more of these postulates (continuity, causality, unitarity, locality, point particles) must be wrong*” (Michio Kaku [KaM])

3. ". . . *we observe that the non standard analysis is presented naturally, within the framework of contemporary mathematics, and thus appears to affirm the existence of all sorts of infinitely entities. . . . it appears to us today that the infinitely small and infinitely large numbers of a non-standard model of Analysis are neither more nor less real than, for example, the standard irrational numbers..."* (A. Robinson, 1966)

4. “..."*We may say a thing is at rest when it has not changed its position between now and then, but there is no ‘then’ in ‘now’, so there is no being at rest. Both motion and rest, then, must necessarily occupy time...."* Aristotle, 350 BC

5. "...*It is probably the last remaining task of the theoretical physics to show us how the term "force" is completely absorbed in the term "number"..."* (R. Taschner, [RTa])

“Real” (point) particles (=real numbers) are the not appropriate physical postulate and should be replaced by ideal particles (=Hyper real numbers), which ***exist*** in mathematics [ARo2]. In the field of Hyperreal numbers only the Archimedian principle (the set of natural numbers is not bounded by a real number) is no longer valid, all other characterizing axioms are the same for boht, the real and the hyper real field.

One way of constructing a system incorporating non-standard reals is to define "numbers" as infinite sequences of reals (or equivalence classes thereof). The state-of-the-art mathematicaly existing quantum objects are the elements of von Neumann’s Hilbert space l(2).

6. ... n*othing stops us to turn over the ration and say that Non-Archimedean adding of quantities is the first cause, and Pseudo-Euclidean space is the model, which reflects this more fundamental ration. ..."* (P.V. Polyan, [PPo]). P. Polyan’s question

> "... d*o the hyperreal numbers exist in the quantum-relative universe*?"

got an answer by M. Heidegger in the form

> “... *in the mathematical physics, ... yes*”.

7. ... *Die MAXWELLschen Gleichungen bestimmen das elektrische Feld, wenn die Verteilung der elektrischen Ladungen und Ströme bekannt ist. Die Gesetze aber, nach denen sich Ströme und Ladungen verhalten, sind uns nicht bekannt. ....(48a), ...der energietensor für die elektromagnetischen Felder ist nur ausserhalb der Elementarteilchen bekannt, ... Phänomenologische Beschreibung der Materie: Hydrodynamische Gleichungen. Wir wissen heute, dass die Materie aus elektrischen elementarteilchen aufgebaut ist, sind aber nicht im Besitz der Feldgesetze, auf welchen die Konstitution jener Elementarteilchen beruht. Wir sind daher genötigt, uns bei der Behandlung der mechanischen Probleme einer ungenauen Beschreibung der Materie zu bedienen, welche der von der klassischen Mechanik verwendeten entspricht. .....(49), ...* (A. Einstein, [AEi]).

8. ... *Tatsächlich haben wir denn auch zweierlei Ar von Gesetzen nötig zur Erklärung der Naturerscheinungen: 1. Die Feldgesetze, gewisse Bindungen des inneren differentiellen Zusammenhangs der möglichen Feldzustände, vermöge deren das Feld allein zur Wirkungsübertragung fähig ist; und 2. Die Gesetze, nach denen die Materie das Feld erregt. Unsere Beschreibung des Feldes, das ein Elektron umgibt, ist erste stammelnde Formulierung derartiger Gesetze. Hier ist das Arbeitsfeld der modernen Physik der Materie, zu welcher vor allem die Tatsachen und Rätsel des Wirkungsquantums gehören*. .... H. Weyl [HWe] §38

9. G.W. Leibniz declined the concept of R. Descartes about the Cartesian continuum of a non-atomistic mechanism, i.e. a continuum assembled by separable particles to enable a homogen and infinite mass continuum. It can exist only a continuum, which is the assembly of non-separable particles; as a consequence the reason/characteristic for such non-separable entities (gr. *monas*) cannot be a quantitative one, but has to be a qualitative (formal) one.

....*Die Vielheit kann nämlich ihre Realität nur von wahrhaftten Einheiten haben, die anderswoher kommen und etwas anderes sind als die Punkte, von denen feststeht, dass aus ihnen das Kontinuum nicht zusammengesetzt werden kann; daher war ich, um diese wirklichen Einheiten zu finden, genötigt, auf ein formales Atom zurückzugreifen, da mein materielles Etwas nicht gleichzeitig materiell (d.h. ausgedehnt) und völlig unteilbar sein kann bzw. ausgestattet mit einer wahrhaften Einheit. Ich musste also die substanziellen Formen, die heutzutage so verschrien sind, zurückrufen und gleichsam rehabilitieren. ...(*Neues System der Natur, 1695, WW IV., p. 478ff).

§2 Mathematical Physics and Mathematics

From Michio Kaku [MKa] we recall:

Because general relativity and quantum mechanics can be derived from a small set of postulates, one or more of these postulates must be wrong. The key must be to drop one or more of these assumptions about Nature on which we have constructed general relativity and quantum mechanics. Over the years several proposals have been made to drop some of our commonsense notions about the universe: continuity, causality, unitarity, locality, point particles.

We will focuss on the later commonsense notation, which is about "point particle" and move back to Newton and Leibniz where the branch seems to be. Before that we recall from Abraham Robinson (Non-standard Analysis, Amsterdam: North-Holland, 1966**):**

"... it appears to us today that the infinitely small and infinitely large numbers of a non-standard model of Analysis are neither more nor less real than, for example, the standard irrational numbers."

From M. Heidegger [MHe2] we recall the „World Picture"

that „***modern physics*** *is called mathematical because, in a remarkable way, it makes use of a quite specific mathematics. But it can proceed mathematically in this way only because, in a deeper sense, it* ***is already itself mathematical***"

The existence of mathematical framework unifying the concepts "particle & wave", already exist, as non-standard models of arithmetic has been proven by Th. Skolem in 1939. Also non -zero infinitesimal small numbers exist in the mathematical world. Ordered fields (like the real numbers, which provides the framwork for standard analysis, including and accepting ir-rational numbers), that have infinitesimal small elements, are called non-Archimedean, i.e. they do not fulfill the Archimedean principle (i.e. the set of natural numbers is not bounded by a real number).

Those numbers are called *Hyperreals* and the related analysis is the **Non-standard Analysis** [MDa].

The question is about the consequences to current physical principles, especially the causality principle, if a (massless) particle with energy (a "photon", modelled as real number) will be replace by hyperreals, i.e. Leibniz monads.

„Purpose" and „causality" are building principles in philosophy (utiliarism, rationalism) and physics (Hamiltonian, Lagrange minimization principle) to give explanations for phenomena.

A phenomenon is seen as the result of a change from some original status to the phenomeneon. The change itself is thereby driven by an (accepted transcendental) concept called „force".

While a „purpose" oriented explanation of a phenomeneon does not need a final route cause, while a causality oriented explanation leads to the question oft he origin cause of everything. In Biology Darwin´s principle is sufficient to explain our human being phenomena just by applying the „purpose" principle.

A „causality" principle to explain the existence of our human species is not needed and all existing explanations there are finally not consistent with the „causality" principle itself, but just religiously to be justified/explained.

We propose to rename the term "purpose" by "***convenience***", which was introduced by M. Heidegger [MHe2] (*). He used the term "Dienlichkeit" (convenience, subservience) in order to describe the feature from which "das Seiende" (be-ings) "is looking to us" The "be-ings" he defined as the entity/unity of substance and form (object and subject, ..., whereby (to M. Heidegger´s opinion) the meaning how both terms are used in modern languages were already the result of a not complete accurate translation from Greek to Latin. The "thing" is a formed substance, i.e. a synthesis of substance and form. The form is assigned/ related to "rationality" and the substance is assigned/ related to "Ir-rationality".

(*) concerning to the above see also our highlighted (blue/bold) text passages of M. Heidegger [MHe] in the reference document (.doc).

"… *modern physics is called mathematical because, in a remarkable way, it makes use of a quite specific mathematics. But it can proceed mathematically in this way only because, in a deeper sense, it is already itself mathematical.* ..."

"... *Mathematical research into nature is not exact because it calculates with precision; rather it must calculate in this way because its adherence to its object-sphere has the character of exactitude. …"*

The existence of non-standard models of arithmetic was discovered by Th. Skolem in 1938/1938, one year after Heidegger´s publication of „The Age of the World". In the mathematical world non-zero infinitesimal small numbers exist, as well. Ordered fields (like the real numbers) that have infinitesimal small elements do not fulfill the Archimedean principle. Such fields are called non-Archimedean. The Non-Archimedean extention of real numbers are the Hyperreals (monads, ideal points) and the related analysis is the Nonstandard Analysis [ARo1/2]. This then closes back the loop to M. Heidegger [MHe2].

§3 Quantum and Gravity Theory

Both, gravitation theory and quantum theory provide an accurate model to describe their “reality” within their own domain. But both theories become inconsistent, when combining them, i.e. a quantum gravitation theory, which unites both, is missing.

This handicap and related unsolved answers and obvious inconsistencies got the branding “particle-wave dualism”, basically based on the philosophy of Descartes.

To overcome current inconsistencies the current string theory approach is, to increase the number of space dimension to enable that each of the 4 elementary forces can in included into the model, without jeopardizing each other and its action on particles, whereby the particles also requires different defintion, depending from the acting force. The approach is therefore to keep all assumptions and extend the model in that way, that all newly to be added elements get integrated in an “orthogonal” way. This concept ended up to a textual desribing “string theory”, using simple mathematical wave models as describing concept to it, which is not a mathematical theory, as “a new, corresponding mathematics” has to be defined/design accordingly.

Our proposal is to challenge, change or skip the one or the other underlying commonsense assumptions about Nature (see [Ka] Kaku M., 1.2) i.e. continuity, *causality*, unitarity, locality, point particles, based on which the quantum and the gravitation theory have been constructed.

From a philosophical perspective ([Fi] Fischer K.) the “particle-wave” “duality” is related to the contradictionary concepts of Plato (“ideal” world) and Aristoteles (“empirical” world). The later one provides our today’s state of the art “to see and define science”. Descartes accepted both “worlds”, but strictly separated between matter and spirit as two different “worlds” without any interacting relations.

Leibniz somehow united and solved Descartes’ “splitted matter and spirit world” into one consistent philosophical model, building on the concept of “monads” ([Fi] Fischer K.). In parallel the monads lead him to the “birth” of the mathematical calculus concept. From a physicist’s perspective at the same time Newton developed the same mathematical concept, where the great concept got its acceptance enabling the classical mechanics. The success of the classical mechanics model, built on the concept of an existing “infinitesimal distance”, resulted into a widely acceptance of “real number” as “real” numbers, even when those “real” numbers include irrational numbers and even when an only small part of the “real” nmbers are rational numbers.

Combining “real” numbers with the mathematical concept of a “function” then enabled the definition of terms like “speed” and “momentum”, which lead the mathematical concept of “**continuity**”.

§4 Action Principles: Causality and Purpose

To describe classical mechanics there are the two great (mathematically equivalent) formalisms: it's the Lagrange (particle mechanics) and Hamiltonian (energy; analytical mechanics) formalism. Both formalisms are from a mathematical point of view "least action principles" or "variational principles" to model (continuous!) motions of particles.

The key observation and "property" of the Hamiltonian formalism is, that the underlying "philosophical" principle of the "least action principle" is not about ***causality***, but about "***<u>purpose</u>***", i.e. minimizing action along an infinitesimal small distance.

The " dx "-concept of differential calculus is only part of Leibniz's solution of Descartes' dualism concept. His "full" solution model, the "monad" concept, got in the meantime a mathematical description ([Ro] A. Robinson et. al), which is the non standard analysis, based on non-standard numbers (hyper-reals), which includes the real numbers. The concept of continuity (as most of all other standard mathematical concepts and theories) keeps being valid in that sense, that the restriction to only real numbers gives the "standard" continuity.

From B. Russell [BRu1], chapter 4 we recall:

"*Activity is to be distinguished from what we mean by causation. Causation is a relation between two phenomena in virtue of which one is succeeded by the other. Activity is a quality of one phenomenon in virtue of which it tends to cause another. Activity is an attribute corresponding to the relation of causality; it is an attribute which must belong to the subject of changing states, in so far as those states are developed out of the nature of the subject itself. It is an actual quality of a substance, forming an element in each state of the substance*."

In current mathematical physics the Hamiltonian and the Lagrange principles are equivalent due to the Legendre transformation. The corresponding mathematical (world) framework are n-manifold. To keep all known forces consistently modeled within this framework it needs to be that each added force into that model increases the dimension of the manifold "world". Current Superstring theory requires the dimension 11. Unfortunately the existing mathematics doesn´t work anymore. For the Superstring theory a to-be-developed mathematic is required. This is a fairly high prize to be paid just to get the 4 nature forces modelled build on current quantum field theory.

Therefore: „why not allow for a moment the following statements:

„Causality" is a specific concept, especially developed by human being only due to Darwin´s principle („survival of the fittest"); „causality" is not at the same level as „learning". Standard Analysis is the appropriate mathematical framework for mathematical physics, which reflects current way of describing human world. „Causality" and physics require experiments and measurements. Measurements applying mathematical arithmetics require the **Archimedean principle**.

„Purpose“ is a more general concept of Nature in the (transcendental) sense of M. Heidegger (“Dienlichkeit”, [MHe2]). It allows also „learning“, and not only specifically for human beings. Non-standard Analysis provides the appropriate mathematical framework for mathematical, non-Archimedean physical field models. It defines the underlying mathematical (and therefore the mathematical physics) existing “thing” of phenomena.

Relations between different physical model formalisms, describing same or similar physical models, are mathematically handled by properly defined transformations. The probably most well known examples are the Lagrange and Hamiltonian formalism. Both are equivalent in the chosen mathematical context of real fields. The Legendre transformation is used to prove this statement. The “real field” context means that the underlying number system are the real numbers and the related physical “objects” are the particles with no physical meaning at all.

A "particle" has no physical meaning and nearly all "real" numbers are from a mathematical perspective “irrational”.

At the same time there is a linkage of the (meta-physical) particle "object" to physical (experimental) observation to describe or/and explain concepts like force, momentum, etc. respectively continuity, differentiability, etc.

On the other side, the "transcendental" Dirac "function" (mathematically well defined) is no “real” function at all, but it is a well accepted object by the physicists. In case of dimension n=1 it is a distribution of the Sobolev Hilbert space $H_{-1/2-\varepsilon}$, for $\varepsilon > 0$.

Funny enough to be mentioned, that no physicist seems to have any problem to accept, that the area spanned by the Dirac "function" (which is the x-axis and the positive y-axis) is equal to 1.

All in all the language used by physicists (which is about using "appropriate mathematical terminologies" put into context to describe/explain "observations"), which is given in a certain mathematical framework ("a given stage"), is already anticipating the expected observations resp. the to be observed or described "objects" ("the played scenes of actors on the "given" stage"), before the experiment/observation happens (i.e. "before the act starts"). The gravitation field equations are THE classical example for such a situation.

As all physical core conceptional assumptions are anyway transcendental we propose to re-build the framework, changing

- physical particles, real points replaced by monades, ideal points

- continuous, differentiable functions replaced by Distributions and Hyperfunctions

- 4-dimensional vector space replaced by quaternions

- 4-dimensional manifolds,11-dim superstrings space replaced by 4-dim. (or 4.5 dimensional) integral currents

- Lagrange formalism replaced by Hamiltonian formalism.

Our idea is therefore basically to replace Newton`s massless (real number) point/particle concept by Leibniz`s (non-standard (ideal) number) "living" ideal point concept and putting the latter one into the context of k-forms with its relation to partial differential equations and field models.

This enables the definition of an appropriate Hilbert space and Hermitean operator framework, which leverage current location & momentum and corresponding eigen differential (with its relation to quantum wave packages and (continuous) spectral theory due to John von Neumann and Paul Dirac)) convergence issues. Werner Heissenberg`s uncertainty principle marks the borderline between standard vs. non-standard Hilbert space, i.e. between the "real" causality oriented and the "ideal" purpose oriented perceived or assumed "world". The objective of this new framework is to enable contiguity in quantum gravitation by an appropriately define "force".

For a first touch and feel related to "Hyper-complex differential form calculus, function theory in quaternions and Clifford algebra (generalized complex analyticity to quaternions by Cauchy-Riemann approach) with weaken classical conditions compared to the classical M-differentiability and complex-analyticity" we refer to R.S. Kraußhar, "A characterization of conformal mappings in R(n) by a formal differentiability condition"

"The exterior derivative d is a closed densely-defined unbounded operator in an appropriate Hilbert space setting, which is connected to the de Rham complex. Due to the Hodge decomposition and the Poincare inequality the mixed (*), weak formulation of the Hodge-Laplacian is well-posed. Leveraging proofs for well-posed PDE in the framework of the calculus of variations might enable an appropriate definition of a mixed formulation of hyperbolic (differential form) gravitation equations, where e.g. well known concepts modelling shock wave singularities/observations can be applied."

As a first conclusion for state-of-the-art mathematical models to describe gravitation theory all „manifolds", where the Legendre transformation shows the equivalence of „purpose" and „causality" are excellence to explain human being world phenomena, but has to fail, when trying to explain quantum gravitation phenomena via „particle-wave dualism"or just simple „time". The „in-between" between „particle-wave" or just „two points in time" seems to fulfill „its" purpose quite successful (at least in the sense of Darwin). There is no real reason for a causality of fit. And mathematics provides the framework for a corresponding „World Picture" with the concept of Hyperreals and Nonstandard Analysis.

Today we think of the set of real numbers as equivalent to the set of points of the real line - a sort of ruler extending endlessly in both directions from the point corresponding to zero. To the ancient Greeks, there were only points corresponding to rational numbers (ratios of whole numbers, e.g., 2/5) and between any two points on a line there were only a finite number of such rational "points". When irrational numbers were discovered, they were deemed "incommensurable", meaning they could not be expressed as such ratios and, in a sense, were non-measurable.

From a mathematical point of view the difference between the field of Hyperreal numbers and real numbers is the missing *Archimedian principle*, which is, that the set of natural numbers is not bounded by a real number. This (missing commosense assumption) might already indicate an apropriate modification of Heissenberg/von Neumann quantum mechanics Hilbert space framework, just by replacing the summation indices out of N by indices out of N*.

As mathematical sophistication increased since Leibniz, the ideas of Cauchy, Weierstrass and others took hold, and monads and moments - in their original guise - faded away. In the Standard Analysis that derived from their work, all real numbers were either rational or irrational, and "infinitesimal" came to mean simply very, very small, but real.

There is an effective limit to the measurability of distances between points that are extremely close together. So, in a sense, there are "spaces" around points in which infinitesimals might reside. Perhaps aspects of logic break down, as they seem to in quantum mechanics, when dealing with microcosmic worlds.

Leibniz's differential calculus, which enabled the mathematical modelling of continuity and continuous functions in combination with Newton's massless particle model in the context of his mechanics models (also to describe gravitation) got a great success. At the very end this approach in combination with Einstein's gravitation theory lead to massless particle, which are purely energy, but still acting as particle, which can only be influenced by independent acting forces on it. On the other side, forces are only measured in such a way, that massless particles are assumed and observed due its behaviour to such forces. In other words: the forces are transcendental, and the massless particles are "real". Applying continuity is in such a framework at the very end not possible, which is in line with Heissenberg's uncertainty relation.

Why not turning the whole thing around, gonig back to the complete idea of Leibniz, which lead him to his differential calculus?

Leibniz proposed monads, which are completely independent with its own "internal" living force. The relations between monads are somehow "self-organizing" via a proposed "pre-defined harmony" concept. In modern terminology this could mean, that the first one describes a real continuity beyong our world, where the border is given by Heissenberg's uncertainty relation (which is valid in the Hilbert-space $L_2 \cong l_2$). The later one proposes instead of "causality" a "purpose", which is responsible for an appropriate "inter-relation" of the living forces (=monads). The definition of non-standard numbers ([Ro]) is very much related to Leibniz's monad concept. It is based on infinite series of numbers with certain properties. Trying to make a link already from this to l_2 might indicate relations of non-standard (hyper-real) numbers to quantum theory "particle" modelling.

Another relation to some basic concepts of standard quantum mechanics might be, roughly speaking, moving from a discrete eigenvalues producing bounded, hermitian and positive definite operator to a bounded, hermitian operator only. The Leibniz concept of the monad is per definition transcendental, i.e. additionally, if one would accept a living force as physical "reality" a continuous eigenvalue spectrum becomes physical relevant (from a non empirical perspective), i.e. the spectral theorem (developed by Hilbert, von Neumann, Dirac) can be applied to (see also §18). This would put an alternatve light for instance on modelling the zero point energy.

§5 A Concept Approach for a Quantum Gravity Model

Our proposal to solve current particle-wave dualism in a hyperbolic Maxwell and Einstein world is the following:

to apply Leibniz’s concept completely and consistently to the Hamiltonian formalism, based on an appropriate extension from “real” to “hyper real” world and based on a new physical principle, which is in line with the key proposition of Leibniz’s philosophy:

1. to extent “particle”/“continuity” to the (trancendent) “hyper real” case

Use “hyper real“ numbers instaed of only “real” numbers to model “particles”, still without any physical extension. As a consequence the definition (!) of “continuity in a standard sense” (which is to the author’s opinion not a philosophical principle, but only a term resp. definition) has to be replaced by “continuity in a non-standard sense” and there is only one type of particle necessary (instead of currently at least two types: Bosons and Fermions.

2. only assume “purpose”, not “causality” for the (trancendent) “hyper real” case

modelled by hyperbolic PDE using variational principles (with its underlying concepts of actual and virtual displacements), i.e. “least action principle”.

This concept proposes

A. a new (explicit transcendental) physical principle ...

There is only one force in a 4 dimensional (non-standard) space-time continuum, which unites the known existing 4 forces in the nature. The underlying source, which is at the same time the "basic" entity=quantum=inflaton is the "monad", which has its "own" force (Leibniz' "vis viva", which is a transcentendal form). The measurable and realized force in our "real" world is a radiation out of the "monad", mathematically described as a specific sherical wave as solution of a hyperbolic radiation equation in a non-standard analysis framework. All, what's within the monade, i.e. beyond the border from "real" to "hyper-real", is "transcendent" in the sense of Kant, but there is still a principle of "purpose" valid, which garantuees the linkage to our world's causality; as this "purpose" get its realization on this side of the border, both is valid, the same purpose, which now is equivalent to causality, i.e. Lagrange formalism becomes valid, too.

B. ... a mathematical program for a consistent model combining quantum (particle) and gravitation (field) theory

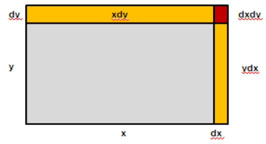
which neccessarily has to unite Maxwell and Einstein equation consistently, overcoming current inconsistencies about nesseccary singular behavior of electronic particle, solving a hyperbolic (radiation) differential equation described by "least action principles" (as Einstein's gravitation equations) in a non-standard analysis framework, modelling a radiation acting out of "monades", which fits to Maxwell's and Einstein's equations. In case this would be successful there then will be a direct link from Maxwell's electron ($U(1)$) (in 4 dimension time-space continuum) to Einstein's graviton (in a 4 dimensional Einstein space), without having to go via $SU(2)\otimes U(1)$ and potential $SU(2)\otimes O(10)$ models, in which case for each step the number of dimensions has to be increased to ensure consistency until eleven, at least. *The field of real numbers R can be interpreted as the Lie algebra of the compact, one-dimensional group $U(1)$, in which the only relevant attribute of an electron within the Maxwell theory – i.e. the electric charge - is modelled. For more complex particles, which requires additional defining attributes (e.g. color, taste), the "standard modelling approach" requires a higher dimensional Lie group (a Lie group is a group, which is at the same time a manifold to allow differential calculus, Lie derivatives are tensor fields which keep invariant under symmetry operations).*

In [RSe], 7.6, some arguments are given, which characterize the space-time continuum with dimension $n = 4$ in relation to the above.

§6 The Legendre Transformation

Leibniz formula is already giving non trivial differential calculus in the form

$$d(x+y) = dx + dy \qquad\qquad d(xy) = xdy + ydx + dxdy = xdy + ydx \ .$$



In standard analysis the term $dxdy$ *is neglected as infinitely small of second order (!). This might be a first opportunity, when extending k-forms into a non standard framework:*

Lagrange --> Hamilton: $L(x,y) \to H(x, \frac{dL}{dy})$

The **Legendre transformation** (Lagrange --> Hamilton) of $f(x,y)$ is defined by

$$g := g(x,y) := \psi y - f = y\psi(x,y) - f(x,y) \text{ and}$$

$$d(g) = yd\psi - \frac{\partial f}{\partial x}dx + (d\psi dy) \ .$$

The product $d\psi dy$ is neglected to be zero in the standard theory as infinitesimal small of second order compared to dx. If one would neglected this and calculate in a non-standard way it would result into

$$d(g) = (y+dy)d\psi - \frac{\partial f}{\partial x}dx \ .$$

Proof: Putting $\psi := \psi(x,y) := \frac{\partial f(x,y)}{\partial y}$ the differential of $f(x,y)$ gives

$$df = \frac{\partial f}{\partial x}dx + \frac{\partial f}{\partial y}dy = \frac{\partial f}{\partial x}dx + \psi dy$$

As holds $\frac{\partial[y\psi(x,y)]}{\partial \psi} = y$ and $d(\psi y) = \frac{\partial(\psi y)}{\partial \psi}d\psi + \frac{\partial(\psi y)}{\partial y}dy = yd\psi + \psi dy$

it follows $d(g) = d(\psi y) - df = yd\psi + \psi dy - \left[\frac{\partial f}{\partial x}dx + \psi dy\right] = yd\psi - \frac{\partial f}{\partial x}dx + (d\psi dy) \ .$

The product $d\psi dy$ is neglected to be zero in the standard theory as infinitesimal small of second order compared to dx.

§7 The Zero Point Energy

In the context of an appropriate tensor analysis for monads the eigendifferentials and wave packages (as currently vehicle to overcome divergent integral issues), which are key concepts in quantum theory, would get new physical interpretations in the context of distribution “functions” acting on “ideal” particles.

The new concept above would be consistent with Huygens’ principles, putting another physical interpretation about the pointwise radiation along front lines and the model of shock waves, providing alternative interpretations of observed diffraction and scattering behaviors.

The new physical principles might overcome current inconsistencies between obervations and quantum field theory projections ging the following explanations:

The zero point energy is proposed to be the radiation out of the “monad”, which is the smallest entity, without any extension and relation to other monads (but “more” than an ideal/real point or a string). Using the word “quantum” for such a “monad” the energy density of such a quantum is called in other context as “quantum vacuum”. It contains all information and all patterns of dynamic energies of the universe.

The Casimir effect shows a zero point radiation. As a request to an appropriate mathematical model the total energy of such a quantum vacuum cannot be divergent.

A photon does not realize any time; how it can act in such a case? As a request to the appropriate mathematical model the asymmetry of “time” in a non-standard hyperbolic world should come out of the specific non-standard spherical “radiation” wave out of the monad.

A. Einstein developed his quantum/photon concept motivated by the question: „if one moves exactly in parallel to a light signal (a photon or a wave?), how the light signal looks like? In principle it should be that the signal of light is a sequence of stationary waves, which are fixed in the time, i.e. the light signal should look like without any movement. If one follows it, it looks like a non-moving, oscillating, electromagnetic field. But something like this seems to be not existed neither caused by observation, nor by the Maxwell-equations model. The later ones exclude the existence of stationary, inelastic waves. Based on the Maxwell equations the electrons would have to loose its energy within nearly no time.

In any relativistic theory the vacuum, the state of lowest energy, if it exists in „reality", has to have the energy zero.

In the same way for any free particle with momentum $\vec{p}$ and mass m the energy has to be

$$E = \sqrt{m^2c^4 + \vec{p}^2c^2}\ .$$

In the literature the ground state energy of the harmonic operator is mostly defined by $\frac{1}{2}\hbar\omega$. Already M. Planck knew that this cannot be, when deriving his radiation formula: he assigned states with n photons the energy $n\hbar\omega$, but not the value

$$(n+\frac{1}{2})\hbar\omega \quad ,$$

which is not compatible with the relativistic co-variant description of photons.

The ground state energy is not measurable. Its chosen value is therefore arbitrarily, triggered only by the fact, to keep calculations as easily as possible, and, mainly, to ensure convergent integrals/series. Energies of freely composed systems should be additive. For photons in a box section (cavity) there are infinite numbers of frequencies ω_i . If one assigns any frequency a ground state energy value $\hbar\omega_i\ /2$, then the ground state energy without photons has the infinite energy

$$\frac{1}{2}\sum_i \hbar\omega_i = \infty\ .$$

The **miss understanding**, that the **ground state energy is fixed** and uniquely defined, starts already in the classical physics: The definition of the Hamiltonian

$$H = \frac{p^2}{2m} + \frac{1}{2}\omega^2 x^2 =: T + V$$

defines the non-measurable ground state energy in that way, that the state of lowest energy, the point $(x=0, p=0)$ in the phase space, that the energy is zero.

§8 Current Super-Gravitation "Theory"

The framework of the super-gravitation theory is a (4+n)-dimensional space. A unification model of the three elementary forces $SU(3), SU(2), U(1)$ are proposed to be by e.g. $SU(5)$ (which can be interpreted also as Kaluza-Klein theory or $O(10)$ or $E(6)$) replacing the combined $SU(3) \otimes SU(2) \otimes U(1)$ symmetry group. This "GUT trial" requires 24 Yang-Mills fields.

The framework of the super-string theory is an 11 dimensional space. Its key concept is modelling the different particles as real extension (with some distance) of virtual points to a string, having different possible vibration modi (which is basically the same, than proposing different energy levels) to differentiate different possible particle types.

Both approaches do not allow a consistent match to the graviton. Both approaches already use implicitely transcendental objects, which are the mathematical points, either as necessary concept of the underlying mathematical model (which is being seen as proof of concept for the physical concept itself) or as neccessary, imaginary point in a physical field, just refer to, in order to desribe the action of a transcendental force of a field.

Instead of this, following Leibniz, why not proposing that there is only one "true" entity (instead of $photons, gluons, leptons, W^{+}, W^{-}, Z^{0} bosons, gravitons,.$), which combines both attributes, "particle" and "living force (vis viva)" "within" one substance, which is the **monad** of Leibniz.

From [Fi] Fischer K. we recall a few statements: *A substance (=monad=points de substance; monas, i.e gr. "entity", "unique entity", "Einheit", "das Eine") is a meta-physical point (points de métaphysiques).*

A physical point requires "extension" to explain mechanics (which is related to the mathematical concept of a metric)*; a mathematical point has no extension, but its real existence is missing* (which is related the mathematical concept of a field)*. The monad fulfills both requirements. The force has to be seen as substance and the substance can only be thought as force. The force is transcendent, i.e. can not be observed and measured; only the action of force is an observable variable. In a pure world of bodies everything is mechanically. Force is therefore a term, which goes beyond this word (fons mechanismi).*

This concept has to be emdedded in the existing (gravitation) field concept, which is based on Einstein's space-time struce, i.e. a 4 dimensional oriented and time-like oriented Lorentz manifold with a twofold, covariant $C^{\infty}-$tensor field g. For every time-like vector x of such a Lorentz space (i.e. for every vector x of the space-time structure, which fulfils $g(x,x) > 0$) the sub-space

$$x^{\perp} := \{ y \in E : g(x,y) = 0 \} ,$$

equipped with the metric $-g$, is euclidical. For two future-oriented vectors x and y of a Lorentz space with $g(x,x) = g(y,y) = 1$ it holds $g(x,y) \geq 1$.

§9 The “tertium non datur” Principle

"tertium datur"=Leibniz`"vis viva" living force of quantums and fields

The "tertium non datur" principle (or the "principium exclusi tertii sive medii inter duo contradictoria") is in contradiction to one of the most successful pieces of the mathematics, the differential calculus (due to Leibniz) modelling changes over time, movements, momentum, Maxwell equations, etc. quite successfully:

in order to see this, a standard example is considering two points on a curve, which can either be the same or they are different. If the "law of the excluded middle" is accepted as true, this are the only two options:

In case the points are different, they define a unique line between each other, which crosses the curve at both points. Therefore this line can never be a tangent to the curve, as a tangent touches a curve at one point only.

In case both points are the same, they define not only one, but infinitely many lines at that point. But which of those should now be the uniquely defined tangent to the curve?

Leibniz solved this contradiction, first starting with the status of two different points and then, at some point in time during his argumentation, they get closer and closer together to become finally identical: this behaviour is explained by Leibniz as an "somehow" existing "infinitely small distance" between both of them.

The mathematican and philosoph L. E. J. Brouwer criticized especially propositions, derived from the law of the excluded middle, of the form

If it holds for no x: not $A(x)$, then for all x: $A(x)$

Alternatively he proposed an intuitional calculus of logic, out of it the law of the excluded middle can not derived.

The negotiation of the law of the excluded middle gets relevant for all propositions related to the infinity and related to past or future events, assuming that truth is ensured knowledge. A popular example for such an proposition is: "Either the world exists without any starting date or it started at some point in time in the past" [LBr]

Core elements of the Cartesian philosophy are matter & mind, extension & idea, life & force. Each pair is complementary, which gets also visible in the today´s wave-particle dualism. Leibniz solved the underlying inconsistency by the concept of "substance", which is active. The activity of substances is metaphysically necessary. A substance is a being capable of actions.

§10 Hyperreal Numbers (Ideal Points)

The set of non-standard numbers $^{*}R$ is a non-archimedean ordered field (the set of complex numbers is an archimedean not-ordered field). *R contains non-trivial infinitely small (infinitesimal) numbers. $^{*}R$ and R have the same cardinality.

The set of non-standard numbers *R is a non-archimedean ordered field (the set of complex numbers is an archimedean not-ordered field). *R contains non-trivial infinitely small (infinitesimal) numbers. The elements *N-N are the infinite natural numbers. The elements *R-R are called hyper-real numbers. An infinite number is greater than any finite number. If a is any finite real number then there exists a uniquely determined standard real number x=st(a), called standard part of a such that r-a is infinitesimal.

In a hyper-real (instead of a today´s ir-rational) world there is no need for a mechanical mass point based (field) model, but it is still possible to explain observed forces modeling action following the "purpose" principle. Our idea is to exchange the transcendental mass point concept (introduced by Newton, modelling observed forces indirectly) by a transcendental „living forces" concept (introduced by Leibniz, enabling a consistent model of the current 4 known forces of the Nature directly with extending the space dimension per each modelled force.

Of course, in the same way as Newton´s physical ideal mass point cannot be measured, Leibniz´ living force (vis viva) can not be measured, too. An appropriate „hyper-real" model has to explain current Nature constants, i.e. when forces are becoming real(ized) at the „toll gate" between purely hyper-real (ity) and real(ity).

The new concept is about proposing Leibniz´s concept of forces (e.g. the "vis viva") as physical principle, replacing Newton´s mass point concept, which is basically replacing today´s concept of (transcendental) particle charges (without any physical meaning) by the living force.

At the same time in a purely hyper-real (ity) the acting forces cannot be explained/ modelled by a "causality" principle (Lagrange formalism) and has to be replaced by a purely "purpose" principle (Hamiltonian´s "action" formalism). In real (ized) observed forces both principles are equivalent (Legendre transform).

Differential equations are used to construct models of reality. Sometimes the reality we are modeling suggests that some solutions of differential equation need not be differentiable. For example the "vibration string" equation (which is the wave equation in an one dimensional space dimension) has a solution u(x,t)=f(x-kt) for any function of one variable f, which has the physical interpretation of a "traveling wave" with "shape" f(x) moving at velocity k. There is no physical reason for the "shape" to be differentiable, but if it is not, the differential equation is not satisfied at some points. But one do not want to throw away physically meaningful solutions because of technicalities." The mathematical framework, which enables such solutions, is the distribution theory.

The continuum hypothesis states that the set of real numbers has minimal possible cardinality which is greater than the cardinality of the set of integers.

The elements $^{*}N - N$ are the infinite natural numbers.

The elements of $^{*}R - R$ are called hyper-real numbers.

An infinite number is greater than any finite number. If a is any finite real number then there exists a uniquely determined standard real number $x = st(a)$, called standard part of a such that r-a is infinitesimal. Differentials dx or dy are infinite elements, while $\frac{dy}{dx}$ is a finite element; the standard number of

$$st(\frac{dy}{dx})$$

corresponds to the derivative of $y'(x)$.

The mathematical building principle for the set of non-standard numbers is based on (internal) set theory and algebra. The core building elements are Frechet filter, ultrafilter, maximal ultrafilter/ideal to construct a field. The physical interpretation of that building principle is that a space-time interval can be divided into a finite number of infinitesimal distances.

The smallest set of propositions (which are analytic) to define real numbers include the Archimedian principle. This principle gives the relation of "real" numbers to the natural numbers. The latter one can be interpreted as a measurement, created by human beings to enable the counting of observed phaenomena. Without that proposition the set fits to the non-standard numbers.

§11 Tensors and k-Forms

From R. Taschner, [RTa] we recall:

"*It is probably the last remaining task of the theoretical physics to show us how the term "force" is completely absorbed in the term "number".*

An alternative mathematical approach for a quantum gravitation theory is proposed embedded in the calculus of variations and the calculus of exterior differential forms based on Robinson`s calculus of hyper-real numbers. The physical rational behind this approach is basically replacing Newton´s transcendental mass point and particle charges by Leibniz´s transcendental ideal points (monads) and living forces, enabling to embed quantum ("particle") theory consistently in a gravitation ("field") theory.

In classical tensor analysis, one never knows what is the range of applicability simply because one is never told what the space is with the known consequences concerning the solution finding of Einstein`s field equations. A analogue picture describing this situation might be a stage and actors, where the stage determines actors` behaviour, but also the other way around. The mathematics of PDE distinguishes between well-posed and not well-posed problems. In this sense the current gravitation field PDE is just shaky.

The difficulties caused by the tensor concept have been overcome in modern times by the theory of differentiable manifolds. Tensor fields do not behave themselves under mappings. With exterior forms one have a really attractive situation in this regard. There is an important inner consistency of the differential calculus, i.e. the exterior derivative of a differential form is independent of the coordinate system in which it is computed. At the same time the differential calculus links perfectly back to Leipniz`s infinitesimal "differentials". Those are transcendental, but nevertheless with a precise mathematical meaning, i.e. those entities are elements of a certain dual space.

How to build now such a space enabling a Hilbert space structure and which dimension it should have?

The first idea is to look at the well known theory of elliptic elasticity problems with its known PDE solution concepts (coerciveness, Korn`s inequality and Garding inequality) and key physical attributes (e.g. torsion).

The basic equations of elasticity are direct consequences of the extended Hamilton's extended principle. A deformable solid body is considered under the influence of two sets of force distributions:

a) so-called body forces F1, F2, F3
b) so-called surface forces T1, T2, T3.

The most usual example for a body-force distribution is the influence of a gravitational field. Surface forces are in operations, whenever a body is subject to contact with external agencies at its surface.
Playing the role of the generalized-force components are the body- and surface-force distributions, which act upon the solid as influences of external agencies. The generalized

coordinates are the components of the displacement u1, u2, u3. The strain potential energy per unit volume is usually described by W. The Hooke's law gives the relation between appropriate derivatives of W to derivatives to T. The second Korn's inequality (which is a Garding type inequality) enables to formulate well-posed elasticity equation problems.

The second idea refers to the geometric interpretation of mass gravitation of the Einstein equations (with its relation to the tensor theory). Those field equations for the empty space-time arises from setting the first variation (with respect to the space-time metric) of the integral of the scalar curvature equal to zero. E. Cartan extended Einstein`s theory by considering torsion and put it into relation with the spin of matter fields. The idea is to work in the framework of the calculus of differential forms, (adapted) moving frames (by which the differential invariants Gauss curvature K and mean curvature H can be derived) and to replace the "scalar curvature minimization" problem in a semi-Riemannian (Lorentz-) manifold (which is always torsions-free) by building an appropriate hyperbolic manifold allowing torsion and moving from a scalar constant-curvature to a scalar constant-mean-curvature. This leads to the concept of ***minimal surface*** in the frameworks of the calculus of ***differential forms***.

§12 Manifolds and Minimal Surfaces

Hamiltonian principle is linked to skew symmetric bilinear forms. In case such bilinear forms are of maximal rank it's called as symplectic form.

The classical geometry of a structure of a 4-manifold is Kaehler geometry, the geometry of a complex manifold with compatible Riemannian metric. In contrast to it, the symplectic geometry is the geometry of a closed skew-symmetric form. It is a 2-dimensional geometry that measures the area of complex curves instead of the length of real curves.

There is an important difference between **Kaehler and symplectic manifold**:

- a Kaehler manifold M has a fixed complex structure built into its points; M is made from pieces of complex Euclidean space C(n) that patched by holomorphic maps. One adds a metric g to this complex manifold and then defines the sympectic form.

- a symplectic manifold first has the form w, and then there is a family of automorphism

$$J : TM \to TM$$

$$J * J = Id ,$$

that turns TM into a complex vector bundle, imposed at the tangent space level (not on the points).

The relation to Functional Analysis is as follows:

the exterior derivative d is a closed densely-defined unbounded operator (which can be interpreted as the counterpart of the momentum operator in quantum mechanics) in an appropriate Hilbert space setting, which is connected to the de Rham complex. Due to the Hodge decomposition and the Poincare inequality the mixed (*), weak formulation of the Hodge-Laplacian is well-posed. Leveraging proofs for well-posed PDE in the framework of the calculus of variations might enable an appropriate definition of a mixed formulation of hyperbolic (differential form) gravitation equations, where e.g. well known concepts modelling shock wave singularities/observations can be applied.

In relation to the quantum mechanics we mention eigen differentials, which correspond to the formalism of wave package modelling. Von Neumann and Dirac developped the spectral theory, where eigen differentials are orthogonal wave packages.

The Problem of Plateau (raised by J.-L. Lagrange) is to prove the existence of a minimal surface bounded by a given contour, e.g. an arbitrary Jordan curve in a n-dimensional Euclidean space ([JDo]). Naturally, an arrangement of knots in the contour will produce coresponding complications in the minimal surface, such as self-intersections and branch points. In this context there is an interesting characterization of the dimension n: the set of all inner branch points is empty for n=2 (Riemann mapping theorem) and n=3, and it is never empty for n>3 ([ROs]). This is one of several evidence arguments (see our paper in the corresponding section), that an n=3 manifold is more likely the appropriate dimension of the solution space for quantum gravitation field equations than the string theory requires to integrate all 4 Nature forces.

§13 Variational Theory

Building an appropriate mathematical variational theory (minimizing) should be guided by „duality" principles, while

- replacing standard Einstein spaces (whereby within a tensor theory framework one is never told what the spaces really are) by a compact 3-manifold M with minimal surface (e.g. embedded in the Euclidean (Hamiltonian formalism enabling?) 6-space), fulfilling proper conditions, which ensures that the mean curvature is zero (instead of the usual torsions freedom situation of each (semi-) Riemannian geometry). The exterior curvature 2-forms $\theta_{i,j}$ on M are related to the Riemann tensor $R_{i,j,k,l}$ by the basis for the 1-forms on M (σ_l) in the form

$$2\theta_{i,j} = \sum R_{i,j,k,l}\sigma_k\sigma_l$$

Potential theory due to W. Hodge might enable such coerciveness conditions, enabling minimal surface properties and unique solutions of the gravitation field equations formulated in a variational theory framework. E. Cartan extended Einstein`s theory by considering torsion and put it into relation with the spin of matter fields. For some research to the latter topic and physical interpretation of torsion in space-time dimensions we refer to [FHe]

- explaining the assymmetry of our world´s time arrow by such a framework

- ensuring consistency to the Huygens principle by such a framework, which is completely neglected by the current string theory.

The mathematical counterpart of the Huygens principle is the Duhamel integral. The Hodge operator gives the d'Alembertian operator in case of Euclidean spaces, the underlying Green function for the (parabolic) heat resp. the (hyperbolic) wave equation (related to the Duhamel integral) are the Gauss-Weierstrass function resp. the Fourier-inverse of the sinc function; singularities caused by the sinc-function might be manageable by the Hilbert-transform trick from the RH proof.

"The exterior derivative d is a closed densely-defined unbounded operator in an appropriate Hilbert space setting, which is connected to the de Rham complex. Due to the Hodge decomposition and the Poincare inequality the mixed (*), weak formulation of the Hodge-Laplacian is well-posed. Leveraging proofs for well-posed PDE in the framework of the calculus of variations might enable an appropriate definition of a mixed formulation of hyperbolic (differential form) gravitation equations, where e.g. well known concepts modelling shock wave singularities/observations can be applied."

§14 k-Forms and Minimal Surfaces

Smooth manifolds can show singularities after a limit process; the missing compactness leads to difficulties, when proving the existence of minimal surfaces; varifolds are a generalization of the comcept of differentiable manifolds by replacing differentiability requirements with those provided by rectificable sets: they can be loosely described as generalized surfaces (manifolds) endowed with multiplicity. In an Euclidean environment a varifold is a positive Radon measure. Varifolds are "currents" without signed orientation, which can still be integrated with unsigned volume integration.

We recall from G. Alberti [GAl] some definitions/terminologies:

The theory of integral currents provides a class of generalized (oriented) surfaces with well-defined notions of boundary and area (called mass), where the existence of minimizer can be proven by direct methods. This class is large enough to have good compactness properties with respect to the topology that makes the mass a lower semi-continuous functional.

From [FAl] we recall:

varifolds of dimension one or two are curves and surfaces defined in Euclidean space in a measure theory way: integral varifolds provide a mathematical model for all soap film and soap bubbles;

- a k-dim. current in R^3 is a linear functional into R, mapping a differential k-form to the integral of that differential over an oriented k-rectificable subset of A of R^3

- a k-dim. varifolds in R^3 is a linear functional into R^+, mapping a differential form to the integral of that differential k-form into the nonnegative real number space R^+.

An exterior algebra of a vector space is the algebra of the wedge (or exterior) product, also called an alternating algebra or an Grassmann algebra. A Grassmann algebra is a associative, anti-commutative algebra with a 1-element. It is a sub algebra of the tensor algebra. The manifolds of differential k-forms build an exterior algebra.

An exterior algebra of a vector space is the algebra of the wedge (or exterior) product, also called an alternating algebra or a Grassmann algebra. A Grassmann algebra is a associative, anti-commutative algebra with a 1-element. It is a sub algebra of the tensor algebra. The manifolds of differential k-forms build an exterior algebra.

Any k-dimensional surface in R^n may be thought of as a k-dim. varifold in R^n. The elements of the weakly topologized space of Radon measures on $R^n xG_{n,k}$, where $G_{n,k}$ is the Grassmann manifold of k-dimensional linear subspaces of R^n, are called k-dim. varifolds on R^n.

The definition of k-dimensional currents closely resembles that of distributions: they are dual of smooth k-forms with compact support. Since every oriented k-dimensional surface defines by integration a linear functional on forms, currents can be regarded as generalized oriented surfaces. As every distribution admits a derivative, so every current admits a boundary.

Differentiable extentions of manifolds (varifolds), which allows non-manifolds points (i.e. manifolds, which may have singularities) are the so-called rectificable sets. Integration over differential form over rectificable set gives an unoriented integral. It can also be regarded as real valued function on the space of differential forms, i.e. as linear functional. At the same time this defines signed measures. Those are called rectificable currents. The varifolds, which are obtained by integration differential forms over a rectificable set are called rectificable varifolds.

In general exterior "measures" (monoton, non-negative, $\sigma -$subadditive) on a set Omega are no measures. If a metric is given on Omega there can be defined a metric, exterior "measure". If such an exterior, metric "measure" is restricted to the sigma-algebra of all measurable subsets of Omega, then is becomes a measure (basically restricted to those sets, by which every other set is "splitted additively". This criteria becomes useful, if a sufficiently large number of sets become measurable. This is the case, when every Borel-set in Omega is measurable. In a metric space there is a natural way to define a special class of metric, exterior "measures", the "Hausdorff-measures".

Signed measures (or electric charge distributions) can have negative volumes. This relates to the generalized oriented surface (the k-dim. currents). Haussdorf "measures" (metric, exterior "measures") are used to define the integration of differential forms over manifolds. For example one can ask for the rectificable set having the smallest Hausdorff measure among all sets having a given boundary in some algebraic topological sense.

The notes above shows, that there is a certain degree of freedom, which is still unused in current Geometric Measure Theory, to define a proper measure on varifolds/currents. As a first proposal we suggest to investigate in an alternative underlying metric:

the usual topology on currents is the weak topology based on a Hausdorff measure in an Euclidean vector space environment, i.e. there is still a degree of freedom to define such a weak topology alternatively (see below). We propose to use "Sobolev"- or "Hoelder-continuous" distribution function with scale factor -1/2 with a correspondingly defined exterior "measure" according to the building principle above. The non-integer scale factor indicates that it might be more appropriate to use the concept of integral currents, than that of integral varifolds. This then might enable the application of spectral and distribution theory providing existence, uniqueness and well posted problem formulations in combination with the Ritz-Galerkin procedure to build such appropriate solutions.

From H. Federer, [HFe], we recall the remarkable property of complex integral currents:

an integral curent (of even dimension) in C(n) (or in a Kaehler manifold), which has a complex tangent space almost everywhere is a minimal current. This means, that a piece of a complex curve L in C^2 is absolutely area minimizing when the boundary dL is fixed, even if the surface L has branch points.

A sufficient condition to Banach spaces to ensure sequence compactness with respect to weak convergence is "reflexibility". For the function spaces L_p this is given for $1 < p < \infty$; for $p = 1$is requires the concept of bounded variation functions, which have only jump-type discontinuities. The related function space is a not separable Banach algebra and the Hausdorff measure are related to it. On the other side every Hilbert space is reflexible due to the representation theorem of Riesz. The Fourier transform of the uniform distribution of unit mass over the unit sphere is given e.g. in [BPe].

In the context of a GUT we especially refer to

- the Hildebrand functional to prove the existence of surfaces with given mean curvature H (see below), i.e.

$$J(u) := a(u,u) - 2F(u) \rightarrow Min$$

with

$$F(u) := (Q(u), u, du) \quad \text{and} \quad H := divQ$$

resp. the corresponding equivalent variational equation formulation

- the recently published paper [BMo]

- the embedding theorem of Nash (surfaces as m-dim. manifold in a n-dim vector space)

- branch points of minimal surfaces and corresponding singular parametrization in case of n>3

- quasi minimal surfaces in pseudo-Riemanian manifolds, [BCh]

- the integral equation method of Theodorsen and Garrick for conformal mapping, [DGa].

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