

An alternative Schrödinger momentum operator enabling a quantum gravity theory

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Abstract

Based on the Hilbert resp. Riesz transform operators we provide an alternative Schrödinger operator P^* with related appropriate domain given by

$$D(P^*) = H_{1/2} = H_1 \otimes H_1^\perp.$$

The corresponding physical states of quantum mechanics are correspondently described by vectors of the Hilbert space

$$H_{-1/2} = H_0 \otimes H_0^\perp.$$

The related location-momentum commutator vanishes for all quantum states $\in H_0$. The complementary closed space H_0^\perp can be interpreted as vacuum quantum state space. As a consequence the Fourier wave concept keeps valid for quantum states $\in H_0$, while it is replaced by the Calderón (admissible) wavelet concept for the vacuum energy space H_1^\perp governed by the wavelet admissibility ($H_{1/2}$ – compatible) condition

$$\int \frac{|\hat{\psi}^2(\xi)|}{|\xi|} d\xi < \infty.$$

The alternative framework is valid for all energy-momentum related differential equations (in its weak corresponding variational representation forms), i.e. all 4 Nature “forces” can be modelled in the same Hilbert space framework with underlying identical space-time continuum and space-time dimension $n = m + 1 = 4$. With respect to the quantum states $\in H_0$ we recall the conjecture “that distortion-free families of progressive, spherical wave of higher levels only exist if and only if the Huygens’ principle is valid and that families of spherical, progressive families only can exist for space-time dimensions $n = m + 1 = 2$ and $n = m + 1 = 4$ ” ([CoR], VI, §10). At the same time the Riesz operators are rotation invariant ([StE]).

As the Hilbert resp. Riesz operators are skew-symmetric the corresponding alternative Lorentz-invariant Schrödinger (Klein-Gordon) equation corresponds to the weak representation the wave equation, newly with respect to the inner product of the $H_{-1/2}$ – Hilbert space.

Our proposed concept replaces a dynamical matter-fields (laws of interaction) world (Yang-Mills field) with a truly continuum world with its own laws, being compatible also with the Einstein field equations. This means, that “the obscure problem of laws of interaction between matter and field does no longer arise. This conception of the world can hardly be described as dynamical any more, since the fields neither generated by nor acting upon an agent spate from the field, but following its own laws is in a quiet continuous flow. It is of the essence of the continuum “. The proposed new framework enables “universal field laws of atomic nuclei and electrons spreading out continuously and being subject to fine fluent changes, where e.g. the mass of an electron derives completely from the accompanying electromagnetic field. Even the atomic nuclei and the electrons are not ultimate unchangeable elements that are pushed back and forth by natural forces acting upon them, but they are themselves spread out continuously and are subject to fine fluent changes”. In other words, “all physical phenomena can be reduced to a simple universal field law (in the form of the Hamiltonian principle)”, ([WeH1] p. 171, appendix).

In quantum theory the Schrödinger momentum operator P is defined by

$$\bar{P} := \frac{1}{i\hbar} P := -\nabla = -\partial \cdot$$

The corresponding commutator property

$$[x, \bar{P}] := x\bar{P} - \bar{P}x = Id$$

leads to the Heisenberg uncertainty inequality, being valid in the corresponding Hilbert space framework L_2 . We propose the following alternative momentum operator in an extended quantum state Hilbert state framework $H_{-1/2}$ building on some (rotation) properties of the Hilbert transformation operator H (resp. on the corresponding properties of the Riesz operators for space dimensions $m > 1$) and the related Symm's log kernel operator A (see appendix):

$$\bar{P}^* := \frac{1}{i\hbar} H[P] := -H[\nabla] = -H[\partial] \cdot$$

Let $u_H := H[u]$ denote the Hilbert transform of $u \in H_\beta$. It holds

Lemma: The alternative Schrödinger quantized momentum operator fulfills the commutator properties

$$([x, \bar{P}^*]u, u)_{-1/2} = (u_H, u)_{-1/2} = \begin{cases} = 0 & u \in H_0 \\ \neq 0 & u \in H_0^\perp = H_{-1/2} - H_0 \end{cases}$$

Proof: From

$$[x, \bar{P}^*]u(x) = u_H(x) - (xH - Hx)u'(x)$$

it follows

$$\begin{aligned} ([x, \bar{P}^*]u, u)_{-1/2} &= (u_H, u)_{-1/2} - ([x, H]u', u)_{-1/2} \\ &= (u_H, u)_{-1/2} - ([x, H][u'], Au)_0 \\ &= (u_H, u)_{-1/2} - ([x, H]A[u'], u)_0 = (u_H, u)_{-1/2} + ([x, H]u_H, u)_0 \end{aligned}$$

Applying the commutator property of the Hilbert transform for functions/distributions with vanishing constant Fourier term then leads to

$$([x, \bar{P}^*]u, u)_{-1/2} = (u_H, u)_{-1/2} \cdot$$

The second property related to the orthogonality between a L_2 – function and its corresponding Hilbert transform then proves the proposition above.

Note 1: The one-dimension Dirac “function” δ is not an element of the Hilbert space $\delta \notin H_{-1/2}$, but $\delta \in H_{-1/2-\varepsilon}$.

Note 2: The alternative momentum operator enables a mathematical model, where “*the obscure problem of laws of interaction between matter and field does not arise. This conception of the world can hardly be described as dynamical any more, since the fields neither generated by nor acting upon an agent spate from the field, but following its own laws is in a quiet continuous flow. It is of the essence of the continuum*”. The text in *italic* is quoted from ([WeH1], p 170 ff).

Note 3: The alternative momentum operator can be applied for all space-time dimension $n=m+1$. For $m>1$ the Hilbert transform is replaced by the Riesz transforms ([StE]). We especially note the “rotation invariance” property of the Riesz transform. *“It can be shown by the means of the wave equation of light (which can be immediately extended to n dimension) that only the space of an odd number of dimensions is the extinction of a candle followed by complete darkness about the candle (within a radius that increases as rapidly as light travels). This, at least, shows up an important inner difference regarding the propagation of effects between even and odd number of dimensions. Those particularly simple and harmonious laws which Maxwell had developed for the electromagnetic field in empty space are invariant with respect to an arbitrary change of the standard unit length at every point, provided the world is four-dimension. This principle of “gauge invariance” holds for no other number of dimensions”*. The text in *italic* is quoted from ([WeH1], p 136 ff). In [WeH2] a theory is provided, which links organically the electric field with the mass field. In this theory the simple and harmonic laws, which can be derived from the Maxwell electric field equations, can only be derived in a four-dimensional world. From Einstein (foundation of relativity theory) we quote: *“the Maxwell equations determine the electromagnetic field based on known distribution of charges and currents. The laws, by which those charges and currents behave, are unknown.”*

Note 4: In the context of Note 3 we recall from [CoR], VI, §10, the conjecture *“that distortion-free families of progressive, spherical wave of higher levels only exist if and only if the Huygens’ principle is valid and that families of spherical, progressive families only can exist for space-time dimensions $n=2$ and $n=4$ ”*.

Note 5: ([ScE] p. 157):

“Atoms or quanta – the counter-spell of old standing, to escape the intricacy of the continuum

Our helplessness vis-à-vis the continuum, reflected a late arrival, it stood godmother to the birth of science – an evil godmother, if you please, like the thirteenth fairy in the tale of the Sleeping Beauty. Her evil spell had for a long time been stemmed by the genial invention of atomism. This explains why atomism has proved so successful and durable and indispensable. It was not happy guess by thinkers who “really did not know anything about it” – it was the powerful counter-spell which naturally cannot be dispensed with as long as the difficulty it is to exorcise survives. By this I will not say that atomism will ever go by the board. Its valuable findings – especially the statistical theory of heat – certainly never will. Nobody can tell the future. Atomism finds itself facing a serious crisis. Atoms – our modern atoms, the ultimate particles – must no longer be regarded as identifiable individuals. This is a stronger deviation from the original idea of an atom than anybody had ever contemplated. We must be prepared for anything.”

The closed space $H_0^\perp = H_{-1/2} - H_0$ can be interpreted as wave package space. It is proposed as an alternative wave package model for the Pauli-Weißkopf multi particle theory overcoming the challenge of negative energy values of the Klein-Gordon Equation (KGE).

The proposed alternative framework overcomes challenges like “for only a sufficiently high number of quanta ...” or vice versa challenges like “for small number of quanta, where the term “particle” gets useless”, building on a distributional wave(let) “function” in the form

$$\psi = \sum_1^\infty a_n \varphi_n + \int a_\lambda \varphi_\lambda d\lambda \cdot$$

The sub-(Hilbert) space $H_0 \subset H_{-1/2}$ “supports” a discrete spectrum while its orthogonal complementary space “supports” the corresponding continuous (ground state energy) spectrum.

The conservation laws for energy and momentum are also valid for quantum theory. Taking Planck's formula

$$E = \hbar\omega = \hbar k^0$$

in the single particle theory as a baseline, then the Lorentz-invariant KGE

$$\psi \in H_2: \quad KG[\psi](x) := \left(\left(\frac{\partial^2}{\partial t^2} - \Delta \right) + \left(\frac{mc}{\hbar} \right)^2 \right) \psi(x) = 0$$

is derived from the Schrödinger equation by the canonical energy and momentum operators mapping into quantum world by

$$E \rightarrow i \frac{\partial}{\partial t}, \quad \vec{p} \rightarrow -i \vec{\nabla}$$

with its related (average) energy equation

$$E^2 = p^2 + m^2 \rightarrow i^2 \frac{\partial^2}{\partial t^2} \psi = i^2 \nabla^2 \psi + m^2 \psi.$$

The energy term E^2 allows negative energy values

$$E = \pm c \sqrt{\vec{p}^2 + (mc)^2}$$

of the (plane) wave solution

$$\psi(x^\mu) = N e^{i\vec{p}\vec{x} - Et}$$

which conflicts to the current definition of the ground state energy. This creates a physical interpretation problem for those values. Pauli-Weißkopf overcame this issue by developing the multi particle theory.

An operator in a Hilbert scale framework is only completely defined in combination with the definition of its corresponding domain. In quantum mechanics the domain for the location operator Q is $D(Q) = L_2 = H_0$, i.e. the corresponding domains for the Schrödinger resp. the Laplace operator are given by $D(P) = H_1$, $D(\Delta) = H_2$.

The weak representations of the location and the momentum operators are defined by

$$\psi \in H_0: (\psi, \chi) \rightarrow (Q\psi, \chi) \quad \forall \chi \in H_0, \quad \psi \in H_1: (\psi, \chi) \rightarrow (P\psi, \chi) \quad \forall \chi \in H_0.$$

Applying the alternative Schrödinger operator definition above in combination with a less regular domain (i.e. interpreting those operators as Pseudo Differential Operators) leads to

$$\begin{aligned} \psi \in H_{-1/2}: (\psi, \chi)_{-1/2} &\rightarrow (Q^* \psi, \chi)_{-1/2} \quad \forall \chi \in H_{-1/2} \\ \psi \in H_{1/2}: (\psi, \chi)_{-1/2} &\rightarrow (P^* \psi, \chi)_{-1/2} \quad \forall \chi \in H_{-1/2}. \end{aligned}$$

The Hilbert and the Riesz transforms are skew-symmetric. Therefore, putting as alternative covariant derivatives

$$\partial_\mu^* := \left(\frac{1}{c} H \left[\frac{\partial}{\partial t} \right], R[\nabla] \right), \quad \partial_\mu^\mu := \left(\frac{1}{c} H \left[\frac{\partial}{\partial t} \right], -R[\nabla] \right)$$

in combination with a reduced domains $H_{1/2}$ leads to

$$\psi \in H_{3/2}: \quad KG^*[\psi](x) := \left(\left(\frac{\partial^2}{\partial t^2} - \Delta^* \right) \right) \psi(x) = 0 \quad \text{resp.} \quad p_\mu^* (p^*)^\mu = 0$$

The corresponding weak variational equation is given by

$$\psi = \psi_1 + \psi_1^\perp \in H_{1/2} = H_1 \otimes H_1^\perp:$$

$$(\ddot{\psi}, \chi)_{-1/2} + (\nabla \psi, \nabla \chi)_{-1/2} = (\ddot{\psi}, \chi)_{-1/2} + (\psi, \chi)_{1/2} = 0 \quad \forall \chi \in H_{1/2}.$$

Now the modified ‘‘Laplace’’ operator (as there is a different domain defined) respectively its related (Dirichlet integral) inner product with respect to the restricted to the subset $H_1^\perp = H_{1/2} - H_1$ anticipates also the ground state matter/energy value m^2 of the classical energy formula above. In other words, both summands $\bar{p}^2 + m^2$ are governed by the (extended) Laplace operator with domain $H_{3/2}$.

Putting $\chi := \dot{\psi}$ gives for the ‘‘energy equation’’

$$\psi = \psi_1 + \psi_1^\perp \in H_{1/2} = H_1 \otimes H_1^\perp:$$

$$\frac{1}{2} \frac{d}{dt} \left\{ \|\dot{\psi}\|_{-1/2}^2 + \|\psi\|_{1/2}^2 \right\} = 0.$$

Note 6: The closed (wave package) space $H_0^\perp = H_{-1/2} - H_0$ also provides an alternative framework to model superconductivity, super-fluids and condensates ([AnJ]). Its related $H_1^\perp = H_{1/2} - H_1$ wavelet package (momentum) space is also proposed in [BrK] to enable a finite energy balance inequality for the 3D non-linear, non-stationary Navier-Stokes Equations (NSE). From a mathematical point of view the central change is about

$$D(P) = H_1 \quad \rightarrow \quad D(P^*) = H_{1/2} = H_1 \otimes H_1^\perp$$

Fourier (H_1) waves

\rightarrow **Fourier (H_1) waves**

\rightarrow **Calderon (admissible H_1^\perp) wavelets**

with the wavelet admissibility ($H_{1/2}$ – compatible) condition

$$\int \frac{|\hat{\psi}^2(\xi)|}{|\xi|} d\xi < \infty$$

In [BrK] a global unique weak $H_{-1/2}$ – solution of the generalized 3D Navier-Stokes initial value problem (for all $v \in H_{-1/2}$)

$$(\dot{u}, v)_{-1/2} + (Au, v)_{-1/2} + (Bu, v)_{-1/2} = 0$$

$$(u(0), v)_{-1/2} = (u_0, v)_{-1/2}$$

is provided, whereby the global boundedness of the energy inequality for the space dimension 3 is a consequence of the crucial Sobolevskii estimate (see also Note 4 above) of the non-linear NSE term ([SoP])

$$\frac{1}{2} \frac{d}{dt} \|u\|_{-1/2}^2 + \|u\|_{1/2}^2 \leq |(Bu, u)_{-1/2}| \leq c \cdot \|u\|_{-1/2} \|u\|_1^2.$$

It enables the a priori estimate

$$\|u(t)\|_{-1/2} \leq \|u_0\|_{-1/2} + \int_0^t \|u\|_1^2(s) ds \leq c \left\{ \|u_0\|_{-1/2} + \|u_0\|_0^2 \right\}.$$

In other words, the standard 3D non-stationary, non-linear NSE is not well posed, which is the negative answer to the corresponding Millennium problem.

The H_0 -based 3D NSE model provides only potential flows to model appropriate boundary layer circulation. The lift of an airfoil in inviscid flow requires circulation in the flow around the airfoil, but a single potential function that is continuous throughout the domain around the airfoil cannot represent a flow with non-zero circulation.

Appendix

Some key properties of the Hilbert transform

$$(Hu)(x) := \lim_{\varepsilon \rightarrow 0} \frac{1}{\pi} \oint_{|x-y|>\varepsilon} \frac{u(y)}{x-y} dy = \frac{1}{\pi} \int_{-\infty}^{\infty} \frac{u(y)}{x-y} dy$$

are given in

Lemma: i) The constant Fourier term vanishes, i.e. $(Hu)_0 = 0$

ii)
$$H(xu(x)) = xH(u(x)) - \frac{1}{\pi} \int_{-\infty}^{\infty} u(y) dy$$

iii) For odd functions it hold $H(xu(x)) = x(Hu)(x)$

iv) If $u, Hu \in L_2$ then u and Hu are orthogonal, i.e.

$$\int_{-\infty}^{\infty} u(y)(Hu)(y) dy = 0$$

v) $\|H\| = 1$, $H^* = -H$, $H^2 = -I$, $H^{-1} = H^3$

vi) $H(f * g) = f * Hg = Hf * g$ $f * g = -Hf * Hg$

vii) If $(\varphi_n)_{n \in \mathbb{N}}$ is an orthogonal system, so it is for the system $(H(\varphi_n))_{n \in \mathbb{N}}$, i.e.

$$(H\varphi_n, H\varphi_n) = -(\varphi_n, H^2\varphi_n) = (\varphi_n, \varphi_n)$$

viii) $\|Hu\|^2 = \|u\|^2$, i.e. if $u \in L_2$, then $Hu \in L_2$.

Proof:

ii) Consider the Hilbert transform of $xu(x)$

$$H(xu(x)) = \frac{1}{\pi} \int_{-\infty}^{\infty} \frac{yu(y)}{x-y} dy \cdot$$

The insertion of a new variable $z = x - y$ yields

$$H(xu(x)) = \frac{1}{\pi} \int_{-\infty}^{\infty} \frac{(x-z)u(x-z)}{z} dz = \frac{1}{\pi} \int_{-\infty}^{\infty} \frac{xu(x-z)}{z} dz - \frac{1}{\pi} \int_{-\infty}^{\infty} \frac{u(x-z)}{z} dz = xH(u(x)) - \frac{1}{\pi} \int_{-\infty}^{\infty} u(y) dy$$

iv)
$$\int_{-\infty}^{\infty} u(y)(Hu)(y) dy = \frac{i}{2\pi} \int_{-\infty}^{\infty} \text{sign}(\omega) |\hat{u}(\omega)|^2 d\omega$$
 whereby $|\hat{u}(\omega)|^2$ is even .

The convolution operator

$$(Au)(x) := -\oint \log 2 \sin \frac{x-y}{2} u(y) dy =: \oint k(x-y)u(y) dy$$

has the following Fourier coefficients

$$(Au)_\nu = k_\nu u_\nu = \frac{1}{2|\nu|} u_\nu \cdot$$

The eigenfunctions of the harmonic quantum oscillator

The n^{th} Hermite polynomial is given by ([SzG])

$$H_n(z) = \frac{2^n}{\sqrt{\pi}} \int_{-\infty}^{\infty} (z-it)^n e^{-t^2} dt \cdot$$

Hermite polynomials have only real zeros all of which are simple. The weighted Hermite polynomials

$$\varphi_n(x) := \frac{e^{-\frac{x^2}{2}} H_n(x)}{\sqrt{2^n n! \sqrt{\pi}}} \quad \text{with} \quad H_n(x) := (-1)^n e^{x^2} \frac{d^n}{dx^n} e^{-x^2}, \quad H_0(x) = 1, \quad H_1(x) = x$$

form a set of orthonormal functions. The relation to the Gauss function is given by

$$f(x) = \pi^{1/4} \varphi_0(\sqrt{2\pi}x) \cdot$$

It holds

$$\varphi_n \in L_2, \quad H\varphi_n \in L_2, \quad (\varphi_n, H\varphi_n) = 0$$

and

$$L_2 := H := \text{span}[\varphi_n(x)] = \text{span}[H(\varphi_n(x))].$$

All zeros of the Mellin transforms of the weighted Hermite polynomials lie on the critical line [BuD]. The proof is based on the recursion formula of the Hermite polynomials in combination with an argument from Polya ([PoG]).

The Hermite polynomials $H_n(x)$ fulfill the recursion formula

$$H_n(\sqrt{2\pi}x) = 2xH_{n-1}(\sqrt{2\pi}x) - (n-1)b_n\varphi_{n-2}(x) - 2(n-1)H_{n-2}(\sqrt{2\pi}x) \cdot$$

Using the abbreviation

$$a_n := \sqrt{\frac{2(n-1)!}{n!}} \quad b_n := \sqrt{\frac{(n-2)!}{n!}}$$

this gives the recursion formula

$$\varphi_n(x) := a_n x \varphi_{n-1}(x) - (n-1)b_n \varphi_{n-2}(x), \quad \varphi_0(x) := \pi^{-1/4} e^{-\frac{x^2}{2}}, \quad \varphi_1(x) := 2^{-1/2} \pi^{-1/4} x e^{-\frac{x^2}{2}}$$

The corresponding recursion formula for the Hilbert transformed Hermite polynomials are given by

$$\hat{\varphi}_n(x) := a_n \left[x \hat{\varphi}_{n-1}(x) - \frac{1}{\pi} \int_{-\infty}^{\infty} \varphi_{n-1}(y) dy \right] - (n-1)b_n \hat{\varphi}_{n-2}(x)$$

$$\hat{\varphi}_0(x) = \pi^{1/4} \int_{-\infty}^{\infty} e^{-\frac{\omega^2}{2}} \sin(ax) d\omega$$

Quotes from Philosophy of Mathematics and Natural Science

Hermann Weyl,
[WeH1]

[WeH1] p. 171:

With Newton, gravitation still appears as an instantaneous action into distance. When only nearby action is considered admissible, ether theories of gravitation arise, which at first however are still under the pressure of the purely mechanical interpretation of nature. Of course Newton too was aware of the difficulty, but declined to “frame hypotheses” about the cause of gravitation (apparently he thought of a non-material transmission by virtue of a “spiritual substance” or of the all-penetrating space filled with the omnipresence of God.) The difficulty was overcome by physical means after Faraday had developed the idea of a field for the electric phenomena. Maxwell found that the field propagates from the centers of excitation not instantaneously but with the velocity of light. Nearby action laws, in the form of differential equations, connect the physical quantities characteristic of matter and field, namely charge and current densities and electrical and magnetic field strengths. The force, which with Newton is not an activity determined by and emanating from a single body k but a bond between two bodies k and k' which join hands across an abyss, is split up into an activity of k (excitation of the field determined by k alone) and suffering of k (temporal change of its momentum caused by that field). Between them the expanse of the field is spread out according to laws of its own of the utmost simplicity and harmony. The field transmits momentum as well as energy from one body to another; a radiating body not only loses energy, but as it radiates light in one direction it recoils in the opposite direction. In the field we therefore have spatially localized energy and momentum. The scalar densities and the components of the vectorial current densities of energy and momentum can be computed by means of simple laws from the two field strengths. The ponderomotoric effect of bodies upon one another is due to an exchange of field energy and momentum against kinetic energy and momentum of matter and vice versa; the increase or decrease in time of total energy or total momentum of any part V of the field is compensated by the current of energy or momentum going through the surface of V . If we determine the center of energy of a portion of space containing both matter and radiation, in the same way as we determine the center of gravity (mass center) of a “ponderable” body, it turns out that the total momentum \vec{I} contained in this portion has the same direction as the velocity \vec{v} of the center of energy. If we set $\vec{I} = m\vec{v}$ the proportionality factor m may well again be called the inert mass. It is connected with the energy E by the universal relation $m = E/c^2$, where c is the velocity of light. A portion of a field such as the radiation in empty space enclosed by massless shell (Hohlraumstrahlung) possesses inert mass like an ordinary body. Thus the strength with which a body, in the face of diverting forces, persists on its natural course as prescribed by the field of inertia depends on the energy compressed in the body. The mass of the electron certainly derives in part from accompanying electromagnetic field. *Or even completely?* Since all physically important properties of an elementary material particle, as we have seen, belong to the surrounding field rather than the substantial nucleus at the field center, the question becomes inevitable whether the existence of such a nucleus is not a presumption that may be completely dispensed with.

This question is answered in the affirmative by the *field theory of matter*. According to the latter a material particle such as an electron is merely a small domain of electrical field within which the field strength assumes enormously high values, indicating that a comparatively huge field energy is concentrated in a very small space. Such an energy knot, which by no means is clearly delineated against the remaining field, propagates through empty space like a water wave across the surface of a lake; there is no such thing as one and the same substance of which the electron consists at all times. Just as the velocity of a water wave is not a substantial but a phase velocity, so that the velocity with which an electron moves is only the velocity of an ideal "center of energy," constructed out of the field distribution. According to this view, there exists but one kind of natural law, namely, field laws of the same transparent nature as Maxwell had established for the electromagnetic field. The obscure problem of laws of interaction between matter and field does not arise. This conception of the world can hardly be described as dynamical any more, since the fields neither generated by nor acting upon an agent spate from the field, but following its own laws is in a quiet continuous flow. It is of the essence of the continuum. Even the atomic nuclei and the electrons are not ultimate unchangeable elements that are pushed back and forth by natural forces acting upon them, but they are themselves spread out continuously and are subject to fine fluent changes. ...

Beside the electromagnetic field we have the metric or gravitation field. The task of merging both into one unit arises.

Auszüge aus

Was ist Materie?

Hermann Weyl,
[WeH]

Seite 28: Aufgrund der angegebenen Tatsachen kommt man notgedrungen zu der Auffassung, daß die Definition „*Kraft = Ableitung des Impulses*“ das Wesen der Kraft nicht richtig wiedergibt. Der wirkliche Sachverhalt ist vielmehr umgekehrt: Die Kraft ist der Ausdruck für eine selbständige, die Körper zufolge ihrer inneren Natur und ihrer gegenseitigen Lage und Bewegungsbeziehung verknüpfende Potenz, welche die zeitliche Änderung des Impulses verursacht

Seite 31: Die Definition des Feldes mit Hilfe seiner ponderomotorischen Wirkung auf einen Probekörper ist nur ein Provisorium. Durch hereinbringen des geladenen Probekörpers stört man immer in etwas das Feld, das es eigentlich zu beobachten galt; befindet es sich im Felde, so gehört er so gut wie die übrigen Konduktoren mit zu den das Feld erzeugenden Ladungen. Das wahre Naturgesetz, das an die Stelle von „*Kraft = Probekörperladung * Feldstärke*“ tritt, wird also anzugeben haben, was für Kräfte das von irgendwie verteilten Ladungen erregte elektrische Feld *auf diese Ladungen selber* ausübt.

Seite 35: Wenn so alle physikalisch wesentlichen Eigenschaften des Elektrons an dem umgebenden Felde und nicht an dem im Feldzentrum steckenden substantiellen Kerne hängen, so muß man sich doch fragen, *ob denn überhaupt die Annahme eines derartigen Kernes nötig ist oder ob wir ihn nicht ganz entbehren können*. Die letzte Frage beantwortet die Feldtheorie der Materie mit Ja; ein Materieteilchen wie das Elektron ist für sie lediglich ein kleines Gebiet des elektrischen Feldes, in welchem die Feldstärke enorm hohe Werte annimmt und wo demnach auf kleinstem Raum eine gewaltige Feldenergie konzentriert ist. Ein solcher Energieknoten, der gegen das übrige Feld keineswegs scharf abgegrenzt ist – der geometrische Begriff des Elektronenradius verliert also seinen präzisen Sinn – pflanzt sich durch den leeren Raum nicht anders fort, wie eine Wasserwelle über die Seefläche fortschreitet; es gibt da nicht ein und dieselbe Substanz, aus der das Elektron zu allen Seiten besteht. Wie die Geschwindigkeit einer Wasserwelle nicht substanziale, sondern Phasengeschwindigkeit ist, so handelt es sich bei der Geschwindigkeit, mit der sich das Elektron bewegt, auch nur um die Geschwindigkeit eines ideellen, aus dem Feldverlauf konstruierten „Energimittelpunktes“. Läßt sich diese Auffassung durchführen, durch welche der die Physik seit FARADAY und MAXWELL beherrschende Dualismus vom Materie und Feld zugunsten des Feldes überwunden wird, so ergäbe sich ein außerordentlich einheitliches Weltbild. Statt der drei Arten von Gesetzen, nach denen das Feld 1. durch die Materie erregt, emittiert wird, 2. sich ausbreitet und 3. auf die Materie wirkt, behalten wir nur die Feldgesetze 2 übrig vom Typus der Maxwellschen Gleichungen, deren Struktur uns völlig durchsichtig ist, während die Gesetze 1 und 3, in deren Dunkel die Physik auch heute noch kaum eingedrungen ist, überflüssig werden. Insbesondere die Gültigkeit der mechanischen Gleichungen gewährleisten durch den aus den Feldgesetzen folgenden differentiellen Energie-Impulssatz, dessen Energiekomponente für das Maxwellsche Feld in Formel (15) angegeben wurde. Man kann dieses Weltbild kaum mehr als ein dynamisches mehr bezeichnen, weil hier das Feld weder von einem dem Felde gegenüberstehenden materiellen Agens erzeugt wird noch auf ein solches wirkt, sondern lediglich, seiner Eigengesetzlichkeit folgend, in einem still kontinuierlichem Fließen begriffen ist. Es ruht ganz und gar im *Kontinuum*, auch die Atomkerne und Elektronen sind keine letzten unveränderlichen, von den angreifenden Naturkräften hin und her geschobenen Elemente, sondern selber stetig ausgebreitet und feinen fließenden Veränderungen unterworfen.

Comparison table: current vs. proposed

	Standard Model of Elementary Particles (SMEP)	Grand Unified Theory (GUT) <i>Proposed mathematical model</i>
Physical model interpretation	all "forces", which interact between different particles, are due to the interaction of related different quantum "types", resp. fields; Standard Model of Elementary Particles by the Yang-Mills field $SU(3) \times SU(2) \times U(1) = SU(3) \times U(2)$,	no single force only "at" big bang and generation of 4 forces during inflation phase; 4 pseudo forces, only, which are measurable actions as a consequence of vacuum state energy as defined per considered variational PDE or PDO
Mathematical framework	additive Gauge fields concept per "force type" in combination with variational principles $G = U(1)$: group of rotations in the plane $G = SU(2)$: the unit of quaternions, Grassman-valued fermion field	single vacuum energy concept for all "pseudo-force" types in combination with variational principles per considered PDE/PDO
Hankel & Hilbert transforms	n/a	$H_v^2 = -H^2 = Id$ H_v, H self-adjoint, L_2 -isometric
Wave and wave package concept	"Dirac" function; its regularity depending from space dimension	distributional Hilbert space element of $H_{-1/2}$, the Hilbert space regularity is independent from the space dimension and better than "Dirac function" regularity for any space dimension
quantum state framework	$H_1 \subset H_0 \subset H_{-N/2-\varepsilon}, \varepsilon > 0$	$H_{1/2} \subset H_0 \subset H_{-1/2}, N > 0$
H_0 polynomial system	Hermite polynomials φ_n whereby φ_1 is the Gaussian function	Hilbert transformed Hermite polynomials φ_n^H : $(\varphi_n, \varphi_n^H) = 0, \hat{\varphi}_n^H(0) = 0$, i.e. the commutator $[x, P] = 0$ for any convolution (singular integral) Pseudo-Differential Operator
$H_{-1/2} = H_0 \oplus H_0^\perp$	n/a	orthogonal projection operator $P_0 : H_{-1/2} \rightarrow H_0$ $(P_0 u, v)_{-1/2} = (u, v)_{-1/2} \forall v \in H_0$ whereby for $u' \in H_{-1/2}$ $(u', v)_{-1/2} = (u, v)_0 \forall v \in H_{-1/2}$; note: the least action principle is equivalent to operator norm minimization problem
Hamiltonian operator(s)	sum of Hamiltonian operators per considered gauge field combinations $H = H_{Dirac} + H_{Yang-Mills} + H_{Higgs} + H_{Einstein}$	single Hamiltonian operator per considered variational PDE/PDO

<p>Harmonic oscillator, Hamiltonian operator</p> $\underline{H} = -\frac{d^2}{dx^2} + x^2$ <p>creation & annihilation operators with different domains</p> <p>H Hilbert transform operator</p>	$\underline{A} = -\frac{d}{dx} + x: H_1 \rightarrow H_0'$ $\underline{A}^* = \frac{d}{dx} + x: H_1 \rightarrow H_0$ $(\underline{A}u, v)_{0'} = (u, \underline{A}^*v)_0 \quad \forall v \in H_0$ <p>\underline{H} self adjoint, eigenvalues real,</p> $\underline{A}^* = -\underline{A}$ $\underline{H} = \underline{A}^* \underline{A} + 1 = \underline{A} \underline{A}^* - 1$	$\underline{a} = -\frac{d}{dx} + x: H_{1/2} \rightarrow H_{-1/2}'$ $\underline{a}^* = \frac{d}{dx} + x: H_{1/2} \rightarrow H_{-1/2}$ $(\underline{a}u, v)_{-1/2'} = (u, \underline{a}^*v)_{-1/2} \quad \forall v \in H_{-1/2}$ <p>$\underline{a}, \underline{a}^*$ are self adjoint, because of</p> $(u', v)_{-1/2'} = (A[u'], v)_0 = (H[u], v)_0$ $= -(u, H[v])_0 = (u, A[-v'])_0 = (u, [-v'])_{-1/2}$
dependency of space dimension	yes	no
space-time dimensions	$N > 10$	4-dimensional scalar field $n = m + 1 = 4$
Poincare "conjecture"	n/a	applicable
Huygens' principle; spherical waves for temporal lines of space-time dimension	n/a	Cauchy & radiation problem for wave equation
particles vs. photons ratio	$6 \cdot 10^{-10}$	particles $\in H_0$, photons $\in H_{-1/2} - H_0$
Planck's action quantum	smallest action quantum, which can be measured in the test space H_0	smallest action quantum, which can be measured in the test space H_0
"time"	<p>eigen-time of a photon is zero, i.e. a photon dies not realize that times goes by; how a photon can act?</p> <p>de Broglie time:</p> $t = \frac{h}{E}$	<p>R. Penrose, The Emperor's New Mind: "the time of our perception does not "really" flow in quite the linear forward-moving way that we perceive it to do. The temporal ordering that we "appear" to perceive is, ..., something that we impose upon our perceptions in order to make sense of them in relation to the uniform forward time-progression of an external physical reality"</p>
Schrödinger momentum operator	$P := -i\hbar\nabla = \frac{\hbar}{i}\nabla$ $H := -\frac{\hbar^2}{2m}\Delta = \frac{1}{2m}\left(\frac{\hbar}{i}\nabla\right)^2$	$D(P) = H_0$: "eigen-functions" are plane-waves $f(x) = e^{ikx}$ which are not defined in H_0 , i.e. $\notin H_0$
<p>Harmonic quantum oscillator (generation & annihilation operators)</p> $H_{osc} = \frac{p^2}{2m} + \frac{1}{2}m\omega^2 q^2 = \frac{1}{2}\hbar\omega(a^+ a + \frac{1}{2})$	$H_{osc} = \frac{1}{2}\hbar\omega(a^+ a + \frac{1}{2})$ $(H_{osc}\psi, \psi)_0 = \infty \quad \forall \psi \in H_0$ $E_n := \hbar\omega(n + \frac{1}{2}), \sum E_n = \infty$	$H = c(\underline{a}^{*2} + \underline{a}^2)$ $(H\psi, \psi)_{-1/2} < \infty \quad \forall \psi \in H_0$
"empty" vacuum	<p>"a closed vacuum space cannot be diluted"</p> <p>(causing inflation/mass generation challenges in the early state of the universe)</p>	<p>no particles in a vacuum, "just"</p> $H_{-1/2} - H_0$ – photons/waves
Heisenberg uncertainty principle	per affected gauge field/Hamiltonian operator	valid in $H_{-1/2}$ not valid in H_0

<p>Schrödinger equation</p>	<p>classical mechanics relationship: continuity equation, Fourier waves</p> $\frac{\partial}{\partial t} \ \psi\ _0^2 = \frac{\hbar}{2im} [(\psi^*, \text{grad}\psi)_0 - (\psi, \text{grad}\psi^*)_0]$	<p>classical mechanics relationship continuity equation, Calderón wavelets</p> $\frac{\partial}{\partial t} \ \psi\ _{-1/2}^2 = \frac{\hbar}{2im} [(\psi^*, \text{grad}\psi)_{-1/2} - (\psi, \text{grad}\psi^*)_{-1/2}]$ $\frac{\partial}{\partial t} \ \psi\ _{-1/2}^2 \equiv \frac{\hbar}{2im} [(\psi^*, \psi)_0 - (\psi, \psi^*)_0] = 0', \quad \forall \psi \in H_0$
<p>Generation of mass of sub-atomic particles and its mass value</p>	<p>energy breaking, phase transition; Lagrange (Klein-Gordon) density and potential of the Higgs field Φ, which is a kind of ether existing throughout the universe</p> $L_{\text{Higgs}} = (D_\mu \Phi)(D^\mu \Phi)^* - V(\Phi)$ $V(\Phi) := \mu^2 \Phi \Phi^* + \lambda (\Phi \Phi^*)^2$ <p>D_μ covariante derivative</p>	<p>phase transition between $H_0 \subset H_{-1/2}$ and the closed sub-space $H_{-1/2} - H_0$</p>
<p>Casimir, Lamb effect</p>	<p>verification in test space H_0</p>	<p>verification in test space H_0</p>
<p>Elementary particles and the options</p> $H := H_0$ <p>or</p> $H := H_{-1/2} = H_0 \oplus H_0^\perp$	<p>Einstein-Podolsky-Rosen paradoxon</p> $H = H_0, \quad H_0^\perp = \{ \}$ <p>gauge/Higgs/(hypothetical) graviton bosons with only symmetric Schrödinger wave function solutions</p> $H = H_0, \quad H_0^\perp = \{ \}$ <p>“matter” particles fermions with only anti-symmetric Schrödinger wave function solutions</p> $\psi = \psi_0 \in H_0$ <p>with $\ \psi_0\ _0 = 1$</p> <p>one therefore gets</p> $\ \psi\ _0 = \ \psi_0\ _0 = 1$	$H = H_{-1/2}$ <p>“wavelet” function/field space</p> H_0 <p>fermions (sub-) space = (matter) test space</p> $\psi = \psi_0 + \psi_0^\perp \in H_{-1/2} = H_0 \otimes H_0^\perp$ <p>with</p> $\ \psi_0\ _0 = 1, \quad \ \psi_0^\perp\ _{-1/2} =: \sigma < \infty$ <p>one therefore gets</p> $\ \psi\ _{-1/2}^2 \leq \sigma + \sum_{i=1}^{\infty} e^{-\sqrt{\lambda_i} \sigma} x_i^2$
<p>Single and double layer potential</p>	<p>Lebesgue integrals,</p> <p>mass density</p> $\mu'(s) = \frac{d\mu}{ds}$ <p>and existing normal derivative with corresponding (mathematically required) regularity assumptions</p> $\frac{dU}{dn}$	<p>Stieltjes integrals,</p> <p>Plemelj’s concepts of an mass element</p> $d\mu_s$ <p>and related “current of a force” on the boundary through the boundary element ds</p> $\bar{U}(\sigma) = - \int_{\sigma_0}^{\sigma} \frac{dU}{dn_s} ds$
<p>Quantum theory of radiation, intensity = surface force density</p> <p>[intensity]</p> <p>= [surface_force_density]</p> <p>= [energy_density]</p>	<p>regularity requirement/assumptions of an existing normal derivative with exterior/interior domain w/o physical meaning</p>	<p>very poor regularity assumptions in the form</p> $\int_{\sigma_0}^{\sigma} d\mu < \infty$ <p>to ensure continuous single layer potential;</p> <p>concept of an apparent size of the element ds as “seen” from the interior point p and double layer potential</p>

$= \frac{N}{cm^2} = \frac{Ncm}{cm^3} = Pascal$		
<p>Maxwell-Lorentz equations, (retarded) potentials and corresponding field equations</p>	<p>Lorentz gauge $A \rightarrow A - \nabla\chi$ $\phi \rightarrow \phi + \frac{\dot{\chi}}{c}$ because of the assumption $H = curlA, \quad curl(E + \frac{\dot{A}}{c}) = 0$ and therefore $E + \frac{\dot{A}}{c} = -\nabla\phi$ Vector field A is not completely determined by the magnetic field H</p>	<p>Vector field A is completely determined by the magnetic field H; no differentiability assumption required; no calibration required due to this purely mathematical assumptions (w/o physical meaning; same situation as for the “differential manifolds” assumptions for the Einstein field equations)</p>

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