

## SUMMARY

This chapter has been concerned with applications of the theory of Chapter 2 to a class of nonlinear boundary-value problems. The associated complementary variational principles were illustrated by various examples taken from mathematical physics. These examples were in (1) hydrodynamics, involving the Liouville and Poisson-Boltzmann equations and the equations of compressible fluid flow, (2) the Thomas-Fermi statistical theory of atomic structure, (3) information theory, dealing with a certain nonlinear integral equation, and (4) the theory of nonlinear electrical networks.

This book is intended to provide an introduction to the theory and applications of complementary variational principles. The treatment was based on local variational theory, leaving several aspects of the subject, such as global theory and numerical topics in the applications, to be developed elsewhere.

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